

[REVISED COURSE]

PR-7295

CON/5092-05.

(3 Hours)

[Total Marks : 100

- N.B. :
- (1) Question No.1 is compulsory.
 - (2) Attempt any four out of remaining six questions.
 - (3) Figures to the right indicate full marks.
 - (4) Assumptions made should be clearly stated.
 - (5) Assume any suitable data wherever required but justify the same.
 - (6) Illustrate answers with sketches wherever required.
 - (7) Answers to the questions should be grouped and written together. i.e. all answers to subquestions of individual questions should be answered one below the other.
 - (8) Use legible handwriting. Use a blue/black ink pen to write answers. Use of pencil should be done only to draw diagrams and graphs.

1. a) A box contains 'a' white balls and 'b' black balls. 'c' balls are drawn at random from the box. (4)
Find the expected number of white balls drawn.
- b) A sample of 100 students is taken from a large population. The mean height of the students (4)
in this sample is 160 cm. Can it be reasonably regarded that, in the population, the mean height is 165 cm and the standard deviation is 10 cm ?
- c) State the uses for the χ^2 test and the conditions for these uses. (4)
- d) Find the least number of ways to choose 6 numbers from 1 to 15 so that all the choices (4)
have the same sum.
- e) If L is a bounded and distributive lattice then prove that complement is unique when it exists (4)

2. a) Find the probability distribution of $X = X_1 + X_2$ where X_1 is the number of spots on the cast (6)
of an unbiased dice and X_2 is the number of heads on four independent tosses of a fair coin.
- b) If 'm' things are distributed among 'a' men and 'b' women, show that the probability (6)
that the number of things received by men is odd is $\frac{1}{2} \left[\frac{(b+a)^m - (b-a)^m}{(b+a)^m} \right]$
- c) Prove the set Q of rational numbers with the binary operation * defined by (8)
 $a*b = a+b-ab$ for $a, b \in Q$ is a semi-group.
Find if it has the identity element and inverse elements. Is it abelian ? Is it a group ?

3. a) If $R(t) = \log M(t)$ where $M(t)$ is the m.g.f. of a discrete random variable then prove that (10)
 $\mu = R'(0)$ and $\sigma^2 = R''(0)$. Hence find the mean and variance of the Poisson distribution.
Also find $P(\mu - 2\sigma < X < \mu + 2\sigma)$ if $\mu = 4$.
- b) Below is given the respective heights x and y of a sample of 12 fathers and their eldest sons (10)

x cm	165	160	170	163	173	158	178	168	173	170	175	180
y cm	173	168	173	165	175	168	173	165	180	170	173	178

Using linear regression estimate son's height if father's height is 172 cm. and estimate father's height if son's height is 173 cm.

Also find the coefficient of correlation between the heights of fathers and sons.

4. a) Prove that $S^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$ is an unbiased estimator of σ^2 . (7)
- b) The mean and standard deviation of height of 8 randomly chosen soldiers are 166.9 cm and (6)
8.29 cm respectively. The corresponding values of 6 randomly chosen sailors are 170.3 cm
and 8.50 cm respectively. Based on this data, can we conclude that the soldiers are, in general,
shorter than the sailors ? Find the 95% confidence limits for the test statistic used.

- c) Consider the set $A = \{0,1,2,3\}$. Give one example for each of the following with justification (7)
- A relation R on A that is neither symmetric nor anti-symmetric.
 - A relation on R that is symmetric and transitive but not reflexive.
 - A binary operation on A that is commutative but not associative.
5. a) Derive Spearman's formula for rank correlation. (6)
- b) The marks obtained by a number of students in a certain subject are approximately normal with mean 65 and variance 25. If 3 students are selected at random from this group, what is the probability that at least one of them would have scored above 75? (6)
- c) Suppose that the length in hours, say X , of a light bulb manufactured by company A is $N(800, 14400)$ and the length of life in hours, say Y , of a light bulb manufactured by company B is $N(850, 2500)$. One bulb is selected from each company and is burned until "death". Find the probability that the length of life of bulb from company A exceeds the length of life of bulb from company B by at least 15 hours. (4)
- d) The probability that an electronic component will fail in less than 1200 hours of continuous use is 0.25. Using normal approximation to binomial distribution, find the probability that among 200 such components fewer than 45 will fail in less than 1200 hours of continuous use. (4)
6. a) $R = \{(1,1), (1,5), (2,2), (2,3), (2,6), (3,2), (3,3), (3,6), (4,4), (5,1), (5,5), (6,2), (6,3), (6,6)\}$ is a relation defined on the set $A = \{1,2,3,4,5,6\}$. Using the matrix of relation show that R is an equivalence relation and find the partition induced by R on A . (5)
- b) Find the smallest equivalence relation containing the two equivalences R and S given below defined on the set $A = \{1,2,3,4,5\}$ (5)
- $$M_R = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \quad \text{and} \quad M_S = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$
- c) Show that (I, \oplus, \otimes) is a ring where $a \oplus b = a+b-i$ and $a \otimes b = a+b-ab$ (10)
Is it an integral domain? Is it a field?
7. a) Four coins are tossed and the number of heads is noted. The experiment is repeated 100 times and the following distribution is obtained (6)
- | | | | | | |
|-------------|---|----|----|----|---|
| No of heads | 0 | 1 | 2 | 3 | 4 |
| Frequency | 7 | 18 | 40 | 31 | 4 |
- Do these results support the assumption of a binomial distribution?
- b) Prove that in a Boolean algebra the following statements are equivalent: (8)
 $a \vee b = b, a \wedge b = a, a' \vee b = I, a \wedge b' = O$
- c) Answer in brief the following with reference to test of hypotheses: (6)
- Why the test statistic is different for large and small samples?
 - What is the necessity of level of significance?
 - What are type I and type II errors and how to minimize them?