T.E / VI / Rev / EXTC / Probability & Random process / Nov / '05 / N.T. EXAM.

PH 98
Con. 4408-05.   PR-4853

(REVISED COURSE)

(3 Hours)   [Total Marks : 100

N.B. Answer any **five** questions.

1. (a) Let B_1, B_2 . . . . B_n be partitions of an event space B_i, i = 1, 2, ... n, for the event B that has occurred. Suppose now an event A occurs. Find expression for P(B|A) in terms of B_i. 10
   (b) Two balanced dices are being rolled simultaneously. If sum of the numbers shown at a time by the two faces is 7. What is the probability that the number shown by one of the face to the dice in this case is 1. 10

2. (a) Explain with sketches how the probabilistic behaviour of a random variable X is defined by its probability density function and its relation with cumulative distribution function, expected value, and variance. 14
   (b) Given $f(x) = ce^{-\alpha|x|}$ and P[| x | < υ]. Find the value of normalization constant C and F [| x | < υ]. 6

3. (a) (i) Explain what is a moment generating function of a random variable. 4
   (ii) If X is a random variable and f(x) is given by $f(x) = f = \frac{1}{b} e^{-(x-a)/b}$, find the first and second moments of X. 8
   (b) If X and Y are independent random variables and z = x + y, find f(z) by the transform method. 8

4. (a) X and Y are random variables. Show that the conditional probability density function of Y, given X = x is given by $f_y(y/x) = \frac{f_{xy}(x,y)}{f_x(x)}$ X and Y are random variables. 10
   (b) Given $f_{xy}(x, y) = \begin{cases} \frac{8}{9}xy, & 1 < x < y < z \\ 0, & \text{Otherwise} \end{cases}$ 10

   Find the marginal density functions of X and Y and conditional densities of Y given X = x, and of X given Y = y.

5. (a) X and Y are random variables. Show that their joint central moment is given by– 8
   $C_{xy}(x, y) = R_{xy}(x, y) - E(X)E(Y)$
   (b) $f_{xy}(x, y) = \begin{bmatrix} 2e^{-x}e^{-y} & , 0 \le y \le x < \infty \\ 0, & \text{elsewhere} \end{bmatrix}$. 12

   Find the correlation, coefficient of X and Y. Are X and Y are independent.

6. (a) If X(t) is an ergodic process show that – 10
   $S_{xx}(w) = \int_{-\infty}^{\infty} R_{xx}(\tau)e^{-jwt}\,d\tau$ where $\tau = t_2 - t_1$ t_1 and t_2 being two instants of time.
   (b) A random process is given by $X(t) = A\cos(\omega_0 t + \theta)$, where A and ω_0 is constant and θ is a random variable uniformly distributed in the interval (– π,– π,). 10
   Determine the power spectrum density of X(t).

7. (a) (i) Explain how a random process can be described by a set of indexed random variables and hence derive expressions for its mean, autocorrelation and autocovariance functions. What will be properties of these functions, if the random process is wide-sense stationa / ? 8
   (ii) Write down the expression of the probability density function if th : process is gaussian. Hence explain how a wide-sense stationary process, if gaussian, is stationary in the strict sense also. 4
   (b) A stationary process is given by– 8
   X(t) = 10 cos [100 t + θ]
   where θ is a random variable with uniform probability distribution in the interval [– π, π]. Show that it is a wide sense stationary process.