S.E (Electrical, Electronics, Instrumentation, Bio-medical and Electronics & Telecommunication) (Sem III) (Rev) Examination October 2006
Applied Mathematics III

Con. 5843-06. (REVISED COURSE) YM-8878

19/06/06

(3 Hours) [Total Marks : 100

N.B. : (1) Question No. 1 is **compulsory**.
(2) Attempt any **four** out of **remaining six** questions.
(3) Answers to the questions should be grouped and written **together**.

1. (a) Use Laplace transforms to find α if $\int_0^\infty e^{-2t} \sin(t+\alpha)\cos(t-\alpha)\,dt = \frac{3}{8}$. **20**
(b) Determine Fourier series for $f(x) = |x|$ in $[-1, 1]$.
(c) Find the map of real axis in z plane under the transformaiton $w = \frac{1}{z+i}$.
(d) Show that the matrix A

$$A = \frac{1}{2}\begin{bmatrix} \sqrt{2} & -i\sqrt{2} & 0 \\ i\sqrt{2} & -\sqrt{2} & 0 \\ 0 & 0 & 2 \end{bmatrix}$$ is unitary. Also write A^{-1}.

2. (a) If A is non singular square matrix of order n. Prove that— **6**
(i) $|\operatorname{adj} A| = |A|^{n-1}$.
(ii) $\operatorname{adj}(\operatorname{adj} A) = |A|^{n-2} A$.
(b) Show that the set up of functions $\left\{\sin\frac{\pi x}{2L}, \sin\frac{3\pi x}{2L}, \sin\frac{5\pi x}{2L} \ldots\right\}$ is orthogonal over [0, L] and construct corresponding set of orthonormal functions. **6**
(c) Find (i) $L\left\{e^{-3t}\int_0^t t\sin 2t\,dt\right\}$ **8**
(ii) $L\left\{\frac{\sin t \, \sin 5t}{t}\right\}$.

3. (a) Obtain half range sine series for $f(x) = \pi x - x^2$ in $[0, \pi]$ and use Parseval's identity to deduce that— **6**

$$\frac{1}{1^6} + \frac{1}{3^6} + \frac{1}{5^6} + \ldots = \frac{\pi^6}{960}.$$

(b) Find a Bilinear transformation which maps the points 0, i, – 2i of z plane onto the points – 4i, ∞, 0 respectively of w plane. Also find the fixed points of the transformation. **6**
(c) Find— $L^{-1}\left\{\frac{1}{5}\tan^{-1}\frac{s+a}{b}\right\}$ **8**

$L^{-1}\left\{e^{-2s}\frac{1}{s^2+8s+25}\right\}$.

4. (a) Determine Non singular Matrices P and Q such that PAQ is in the normal form. **6**

$$A = \begin{bmatrix} 3 & 2 & -1 & 5 \\ 5 & 1 & 4 & -2 \\ 1 & -4 & 11 & -19 \end{bmatrix}$$

4. (a) Determine Non singular Matrices P and Q such that PAQ is in the normal form. 6

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(b) If $f(z) = u + iv$ is analytic function show that $\left[\frac{\partial |f(z)|}{\partial x}\right]^2 + \left[\frac{\partial |f(z)|}{\partial y}\right]^2 = |f'(x)|^2$. 6

(c) Determine Fourier series for $f(x) = x \sin x$ in $[-\pi, \pi]$ and deduce that— 8

$$\frac{\pi - 2}{4} = \frac{1}{1\cdot 3} - \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} - \frac{1}{7\cdot 9} \ldots\ldots$$

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5. (a) Find L^{-1} by convolution Theorem $L^{-1}\left\{\dfrac{1}{(s+3)\left(s^2+2s+2\right)}\right\}$ **6**

(b) Find an analytic function $f(z) = u + iv$ using Milne Thomson method. **6**

$$u + v = \frac{\sin 2x}{\cosh 2y - \cos 2x}.$$

(c) State True or False with proper justification. **8**

(i) Skew symmetric matrix of odd order is singular.

(ii) If $A = [aij]_{3 \times 3}$ where $aij = \frac{i}{j}$ then rank of A is 3.

6. (a) Find the value of λ for which the following system of equations have non zero solution. Solve the equation— **6**

$$x + 2y + 3z = \lambda x$$
$$3x + y + 2z = \lambda y$$
$$2x + 3y + z = \lambda z.$$

(b) If f(t) is periodic function with period 2a, defined by— **6**

$$f(t) = 1 \qquad 0 \leq t < a$$
$$= -1 \qquad a < t \leq 2a$$

Prove that $L\{F(t)\} = \dfrac{1}{5}\tanh\left(\dfrac{a^5}{2}\right).$

(c) Prove the following— **8**

(i) If f(z) and $f(\bar{z})$ are both Analytic functions then f(z) is a constant function.

(ii) The image of rectangular hyperbola $x^2 - y^2 = 1$ under the transformation $w = \frac{1}{z}$ is the Leminiscate $\rho^2 = \cos 2\phi$.

7. (a) Express the function $f(x) = e^{-x}$ as Fourier sine integral where $x > 0$ and prove that— **6**

$$\int_0^\infty \frac{\cos \lambda x}{1+\lambda^2}\, d\lambda = \frac{-\pi}{2} e^{-x}.$$

(b) Find the inverses of— **6**

$$A = \begin{bmatrix} 1 & a & 0 \\ 0 & 1 & a \\ 0 & 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 & 0 \\ b & 1 & 0 \\ 0 & b & 1 \end{bmatrix}$$

and hence find the inverse of—

$$C = \begin{bmatrix} 1+ab & a & 0 \\ b & 1+ab & a \\ 0 & b & 1 \end{bmatrix}$$

(c) Solve the differential Equation using Laplace transforms. **8**

$$\frac{d^2y}{dt^2} + y = t \qquad y(0) = 1, \quad y'(0) = 0$$