

(3 Hours)

[Total Marks : 100

INSTRUCTIONS:

- 1) Question number 1 is compulsory
- 2) Answer any four out of the remaining six questions.
- 3) Figures to the right indicate full marks.
- 4) Write answers to sub-questions of main question collectively together.
- 5) Use of statistical tables is permitted.

- 1.a) Two populations have the same mean but the standard deviation of one is twice that of the other. Show that in samples, each of size 500, drawn under simple random conditions, the difference of the means will, in all probability, not exceed 0.3σ , where σ is the smaller standard deviation. (5)
- b) Given the following information about the marks of 60 students estimate the marks of a student in Mathematics who scored 60 marks in Physics, and the marks of a student in Physics who scored 70 in Mathematics: (5)

	Mathematics	Physics
Mean	80	50
Standard deviation	15	10
Correlation coefficient r	0.4	

- c) Based on the data below determine if there is relation between literacy and smoking: (5)

	Smokers	Non-smokers
Literates	83	57
Illiterates	45	68

- d) Use the dual simplex method to solve the following LPP: (5)

Minimize $z = 2x_1 + 2x_2 + 4x_3$

subject to $2x_1 + 3x_2 + 5x_3 \geq 2$

$3x_1 + x_2 + 7x_3 \leq 3$

$x_1 + 4x_2 + 6x_3 \leq 5$

$x_1, x_2, x_3 \geq 0$

- 2.a) If the following distribution of a discrete random variable X has mean = 16 then find m, n and the variance of X. (6)

x	8	12	16	20	24
P(X=x)	1/8	m	n	1/4	1/12

- b) Seven coins are tossed and the number of heads obtained noted. The experiment is repeated 128 times and the following distribution is obtained:

No. of heads	0	1	2	3	4	5	6	7	Total
Frequency	7	6	19	35	30	23	7	1	128

Fit a binomial distribution to this data i) if the coins are unbiased (7)

ii) if the nature of the coins is not known. (7)

3.a) There are 400 students in the first year class of an engineering college. The probability that any student requires a copy of a particular Mathematics book from the college library on any day is 0.1. How many copies of the book should be kept in the library so that the probability may be greater than 0.95 that none of the students requiring a copy from the library has to come back disappointed? (4)

b) Solve the following LPP

$$\text{Minimize } z = 24x_1 + 30x_2$$

$$\text{subject to } 2x_1 + 3x_2 \geq 10$$

$$4x_1 + 9x_2 \geq 15$$

$$6x_1 + 6x_2 \geq 20$$

$$x_1, x_2 \geq 0$$

i) using the graphical method (4)

ii) using the Big-M method (6)

iii) using the dual of the given LPP (6)

4.a). Two samples drawn from two different populations gave the following results: (4)

	Size	Mean	Standard deviation
Sample 1	400	124	14
Sample 2	250	120	12

Find the 95% confidence limits for the difference of the population means.

b) The mid-term examination marks X and the final examination marks Y of 12 students in the subject of Statistics are given below:

X	71	49	80	73	93	85	58	82	64	32	87	80
Y	83	62	76	77	89	74	48	78	76	51	73	89

i) Find the equation of the least-squares line which will enable one to predict a student's final examination marks on the basis of the mid-term examination marks. (6)

ii) Find the rank correlation coefficient between the marks of the two examinations. (4)

iii) Find if there is improvement in marks from the mid-term to the finals. (6)

5.a) If X and Y are random variables with the same standard deviation σ and zero correlation then show that $U = X \cos \alpha + Y \sin \alpha$ and $V = X \sin \alpha - Y \cos \alpha$ has zero covariance. (6)

b) For 10 pairs of values of x and y the following values are determined: (6)

	x	y
Mean	30.1	47.8
Standard deviation	6.2	9.5
Correlation coefficient r	0.72	

Later on it was found that one pair of values was taken as (34, 47) instead of (43, 74). Determine the correct value of the coefficient of correlation.

- c) Samples of two types of electric bulbs were tested for length of life and the following data were obtained: (8)

	No. of samples	Mean	Standard deviation
Type I	8	1134	35
Type II	7	1024	40

- i) Show that at 5% LOS the difference in the sample means is significant
 ii) Test at 1% LOS whether type I is better than type II

- 6.a) The amount of pollutant X released by an industry should lie between 30 and 35. (6)
 Assume that X is normally distributed with mean $\mu = 33$ and standard deviation $\sigma = 3$.
 The industry gets a profit of Rs. 100 when $30 < X < 35$; Rs. 50 when $25 < X < 30$ or $35 < X < 40$;
 and incurs a fine of Rs. 60 otherwise. Determine the expected profit for the industry.

- b) Tests made on the breaking strength of 10 pieces of a metal gave the following results: (6)
 578, 572, 570, 568, 572, 570, 570, 572, 596 and 584 kg.
 Test if the mean breaking strength of the metal wire can be assumed to be 577 kg.

- c) What is sampling distribution? Distinguish between estimation and testing of hypothesis. (2)

- d) Find all the basic solutions of the following LPP and classify them as (6)
 feasible/ infeasible, degenerate/non-degenerate and optimal/non-optimal:

$$\text{Maximize } z = 2x_1 + 3x_2$$

$$\text{subject to } x_1 + 3x_2 \leq 6$$

$$3x_1 + 2x_2 \leq 6$$

$$x_1, x_2 \geq 0$$

- 7.a) For the Poisson distribution prove that the mean and variance are equal. (5)

- b) i) Using the Lagrangean method solve the NLPP (8)

$$\text{Maximize } z = x_1^2 + 2x_2^2 + x_3^2$$

$$\text{subject to } 2x_1 + x_2 + 2x_3 = 30$$

$$x_1, x_2, x_3 \geq 0$$

- ii) Using the Kuhn-Tucker conditions solve the NLPP

$$\text{Maximize } z = 3x_1^2 + 14x_1x_2 - 8x_2^2$$

$$\text{subject to } 3x_1 + 6x_2 \leq 72$$

$$x_1, x_2 \geq 0$$

(8)