(3 Hours) [Total Marks : 100

N.B. (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions out of remaining **six** questions.
(3) **Figures** to the **right** indicate **full** marks.

1. (a) Evaluate by using L.T. **20**

$$\int_0^\infty \frac{\cos 6t - \cos 4t}{t}\, dt .$$

(b) If $f(x) = c_1\,\phi_1(x) + c_2\,\phi_2(x) + c_3\,\phi_3(x)$ where c_1, c_2, c_3 are constants, and $\phi_1\ \phi_2\ \phi_3$ are orthonormal sets on (a, b)
Show that—

$$\int_a^b [\,f(x)\,]^2\, dx = c_1^2 + c_2^2 + c_3^2$$

(c) Find the constants a, b, c, d, e, if
$F(z) = (ax^4 + bx^2y^2 + cy^4 + dx^2 - 2y^2) + i(4x^3y - exy^3 + 4xy)$ are analytic.

(d) Show that every square matrix A can be uniquely expressed as P + iQ when P and Q are Hermitian matrices.

2. (a) If $A = \begin{bmatrix} 3 & 2 & 2 \\ 1 & 3 & 1 \\ 5 & 3 & 4 \end{bmatrix}$ **6**

Find adj A, A^{-1} hence find B such that :

$$AB = \begin{bmatrix} 3 & 4 & 2 \\ 1 & 6 & 1 \\ 5 & 6 & 4 \end{bmatrix}$$

(b) Find non singular matrices P and Q such that PAQ is in normal form also find its rank **6**

$$A = \begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

(c) Find (i) $L\left\{ e^{-3t} \int_0^t t \sin 3t\, dt \right\}$ **8**

(ii) $L\left\{ \frac{\sin t \cdot \sin 5t}{t} \right\}$.

3. (a) Find Fourier Series for **6**

$$f(x) = \begin{cases} \pi x & 0 \le x \le 1 \\ \pi(2 - x) & 1 \le x \le 2 \end{cases}$$

hence deduce that—

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} \ldots\ldots = \frac{\pi^2}{8}$$

(b) (i) $L^{-1}\left\{ \frac{s^2}{(s^2 + a^2)^2} \right\}$ by convolution theorem **6**

(ii) $L^{-1}\left\{ \log\left(\frac{s^2 + a^2}{s^2 + b^2} \right) \right\}$

(c) Use L.T. to solve **8**
$(D^2 - 3D + 2)\, y = 4e^{2t}$ at $y(0) = -3,\ y'(0) = 5$.

[TURN OVER

4. (a) Find Fourier Series for $f(x) = |\sin x|$ in $(-\pi, \pi)$. 6

(b) Obtain complex form of Fourier Series for $f(x) = \cos h\ 3x + \sin h\ 3x$ in $(-3, 3)$ 6

(c) Find the bilinear transformation which maps the points $2, i-2$ onto the points $1, i, -1$ hence find its fixed points. 8

5. (a) If 6

$$N = \begin{bmatrix} 0 & 1+2i \\ -1+2i & 0 \end{bmatrix} \text{ then}$$

Show that $(I - N)(I + N)^{-1}$ is a unitary matrix.

(b) Find an analytic function $w = f(z)$ whose imaginary part $V = x^2 - y^2 + \frac{x}{x^2+y^2}$ and determine real part u. 6

(c) (i) Find the image of $|z - a_i| = a$ under the transformation $w = \frac{1}{z}$. 8

(ii) Find the orthogonal trajectories of the family of curves $e^x \cos y - xy = c$.

6. (a) Express the function in Hearisides unit step frequency and hence find L-T. 6

$$\begin{aligned} F(f) &= \cos t && 0 < t < \pi \\ &= \cos 2t && \pi < t < 2\pi \\ &= \cos 3t && t > 2\pi \end{aligned}$$

(b) If f(t) is a Periodic function of Period T show that : 4

$$L\{f(t)\} = \frac{1}{1 - e^{-ST}} \int_0^T e^{-st} f(t)\, dt$$

(c) Test the consistency of the following equations and solve them if they are consistent. 6

$$\begin{aligned} x - 2y + 3t &= 2 \\ 2x + y + z + t &= +4 \\ 4x - 3y + z + 7t &= 8 \end{aligned}$$

(d) Find the matrix A if— 4

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix}.$$

7. (a) Obtain half range sine series for— 6

$f(x) = l x - x^2 \quad 0 < x < l$ and use Parseval's identity to deduce that—

$$\frac{\pi 6}{960} = \frac{1}{1^6} + \frac{1}{3^6} + \frac{1}{5^6} + \ldots\ldots$$

(b) Find Fourier cosine integral for— 6

$$\begin{aligned} f(x) &= 1 - x^2 && 0 \le x \le 1 \\ &= 0 && x > 1 \end{aligned}$$

(c) Show that $u = e^x (x \cos y - y \sin y)$ is harmonic. Find the harmonic conjugate V and the analytic function $F(z)$. 8
