

- N.B. :** (1) Question No 1 is **compulsory**.  
 (2) Attempt any **four** questions out of remaining **six** questions.  
 (3) Assumption made should be **clearly** stated.  
 (4) **Figures** to the right indicate **full marks**.

1. (a) Use Induction to prove that if  $F_n$  is the  $n^{\text{th}}$  fibonacci number, then—

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$$F_n = \frac{1}{\sqrt{5}} \left[ \left( \frac{1+\sqrt{5}}{2} \right)^{n+1} - \left( \frac{1-\sqrt{5}}{2} \right)^{n+1} \right]$$

for all integers  $n \geq 0$ .

- (b) In a Survey of 260 college students, the following data were obtained :—

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- 64 had taken a Mathematics Course,
- 94 had taken a Computer Science Course,
- 58 had taken a Business Course,
- 28 had taken both Mathematics and a Business Course,
- 26 had taken both Mathematics and Computer Science Course,
- 22 had taken both Computer Science and a Business Course and
- 14 had taken all three types of courses.

- (i) How many students were Surveyed who had taken none of the three types of Courses ?
- (ii) Of the students Surveyed, how many had taken only a Computer Science Course ?

- (c) Suppose that a man hiked 6 miles the first hour and 4 miles the twelfth hour and hiked a total of 71 miles in 12 hours. Prove that he must have hiked at least 12 miles within a certain period of two consecutive hours. (Prove by contradiction).

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- (d) If  $f$  is the mod-12 function, compute each of the following :—

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- (i)  $f(1259 + 743)$     (ii)  $f(1259) + f(743)$ ,    (iii)  $f(2 \cdot 319)$ ,    (iv)  $2 \cdot f(319)$ .

2. (a) Show that the  $(3, 7)$  encoding function  $e : B^3 \otimes B^7$  defined by—

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$$e(000) = 0000000$$

$$e(001) = 0010110$$

$$e(010) = 0101000$$

$$e(011) = 0111110$$

$$e(100) = 1000101$$

$$e(101) = 1010011$$

$$e(110) = 1101101$$

$$e(111) = 1111011$$

is a group code.

- (b) Define a pigeonhole principle and Let  $T$  be an equilateral triangle whose sides are of length 1 unit. Show that if any five points are chosen lying on or inside the triangle, then two of them must be no more than  $\frac{1}{2}$  unit apart.

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- (c) Prove that if  $x$  is a rational number and  $y$  is an irrational number, then  $x + y$  is an irrational number.

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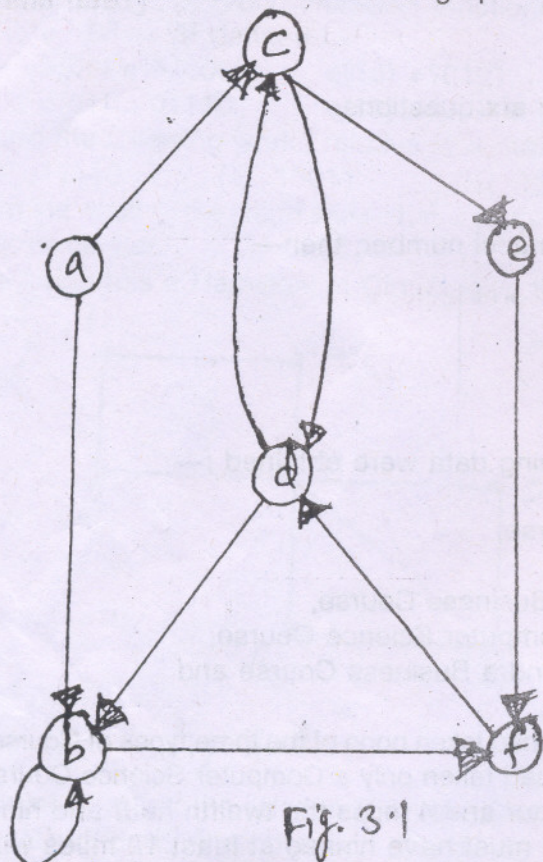
- (d) Find the complete solution of the recurrence relation :—

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$$a_n + 2 a_{n-1} = n + 3 \text{ for } n \geq 1 \text{ and with } a_0 = 3.$$

3. (a) Let R be the relation whose diagram is given in figure 3-1.

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- (i) Find all path of length 3,
- (ii) Find  $MR^2$ ,
- (iii) Find  $MR^\infty$ ,
- (iv) Find  $R^\infty$ .

(b) Let  $P(n)$  be the statement,  $n^2 + n$  is an odd number for  $n \in \mathbb{Z}^+$ .

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- (i) prove that  $P(k) \Rightarrow P(k + 1)$  is a tautology,
- (ii) Is  $P(n)$  true for all  $n$ ? Explain.

(c) Prove by induction that if a relation R on a set A is symmetric, then  $R^n$  is symmetric for  $n \geq 1$ .

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(d) Define a relation on the set  $\{ a, b, c, d \}$  that is—

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- (i) reflexive and symmetric, but not transitive,
- (ii) reflexive and transitive, but not symmetric,
- (iii) transitive, reflexive and symmetric,
- (iv) a symmetric and transitive.

4. (a) Let  $A = \{ 1, 2, 3, 5, 6, 10, 15, 30 \}$  and consider the partial order  $\leq$  of divisibility on A. That is, define  $a \leq b$  to mean that  $a|b$ . Let  $A' = P(S)$ , where  $S = \{ e, f, g \}$ , be the poset with partial order  $\subseteq$ . Show that  $(A, \leq)$  and  $(A', \subseteq)$  are isomorphic.

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(b) Let R be the relation whose matrix is—

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$$\begin{bmatrix}
 1 & 0 & 0 & 1 & 1 \\
 0 & 0 & 1 & 0 & 1 \\
 1 & 1 & 1 & 0 & 0 \\
 0 & 1 & 1 & 0 & 0 \\
 0 & 0 & 1 & 0 & 1
 \end{bmatrix}$$

- (i) Find the reflexive closure of R,
- (ii) Find the symmetric closure of R.

(c) Let  $A = \{ 1, 2, 3, 4 \}$  and Let  $R$  and  $S$  be the relations on  $A$  described by—

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$$M_R = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \text{ and } M_S = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Use Warshall's algorithm to Compute the transitive closure of  $R \cup S$ .

(d) Draw the Hasse diagram of a topological sorting of the given poset—

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(i)

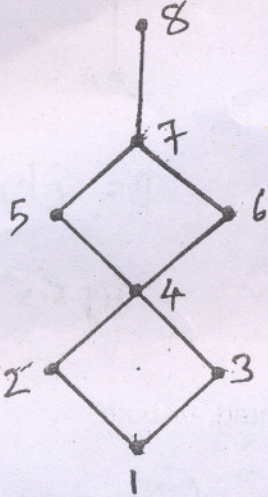


fig. 4.1

(ii)

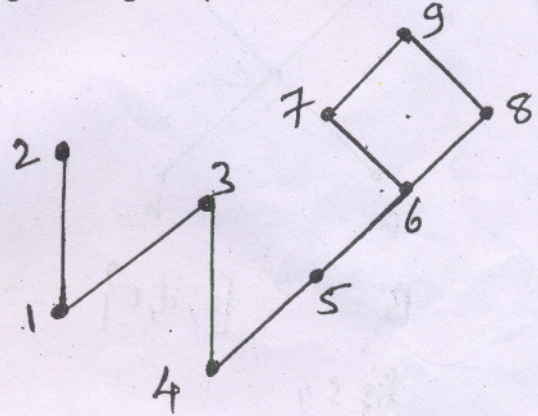


Fig. 4.2

5. (a) Determine the greatest and least Elements, if they exist, of the poset.

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(i)

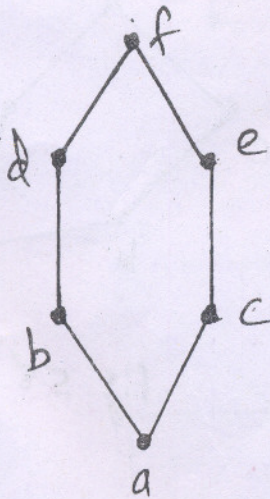
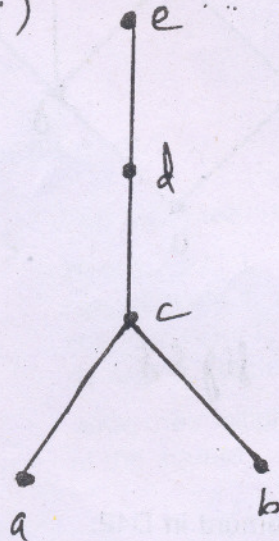


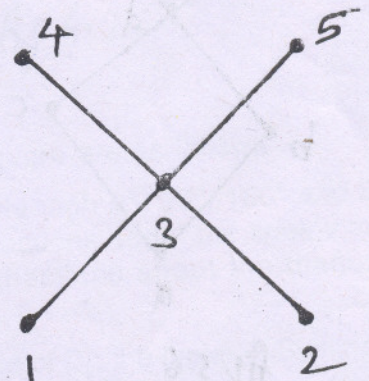
fig 5.1

(ii)



figs. 5.2

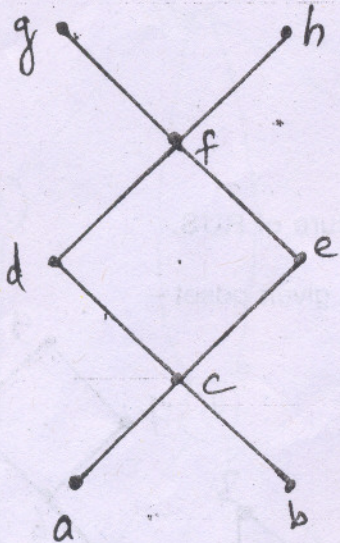
(iii)



figs 5.3

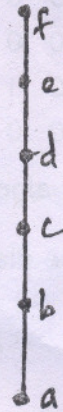
(b) Find if they exist—

- (i) all upper and lower bound of B
- (ii) the least upper and greatest lower bound of B.



$$B = \{c, d, e\}$$

fig. 5-4



$$B = \{b, c, d\}$$

fig. 5-5

(c) Determine whether each lattice is distributive, complemented, or both.

(i)

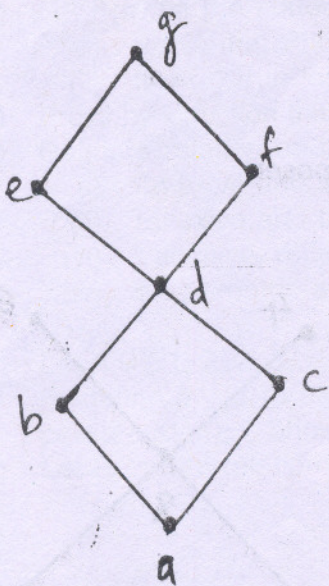


fig. 5-6

(ii)

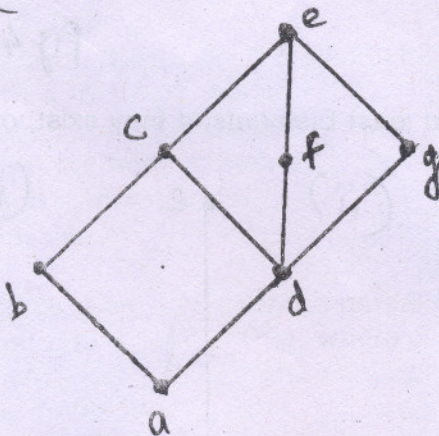


fig. 5-7

(iii)

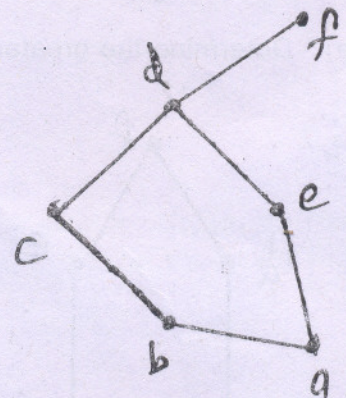


fig. 5-8

(d) Find the complement of each Element in D42.

6. (a) We are given the ring  $\langle \{ a, b, c, d \}, +, \cdot \rangle$  whose operations are given by the following table :— 6

+	a	b	c	d
a	a	b	c	d
b	b	c	d	a
c	c	d	a	b
d	d	a	b	c

·	a	b	c	d
a	a	a	a	a
b	a	c	a	c
c	a	a	a	a
d	a	c	a	a

Is it a commutative ring ?  
 Does it have an identity ?  
 What is the zero of this ring ?  
 Find the additive inverse of each of its element.

(b) Let  $A = \{ a, b, c, d \}$  and  $r$  be a relation on  $A$  whose matrix is— 6

$$M_R = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(i) Prove that  $R$  is a partial order, (ii) Draw the Hasse diagram of  $R$ .

(c) Let  $A = B = R$ , the set of real numbers 4

Let  $f : A \rightarrow B$  be given by the formula  $f(x) = 2x^3 - 1$  and Let  $g : B \rightarrow A$  be given by—

$$g(y) = \sqrt[3]{\frac{1}{2}y + \frac{1}{2}}$$

Show that  $f$  is a bijection between  $A$  and  $B$  and  $g$  is a bijection between  $B$  and  $A$ .

(d) Show that the graph given is planar. 4

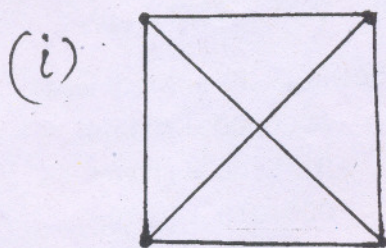


Fig 6.1

(ii)

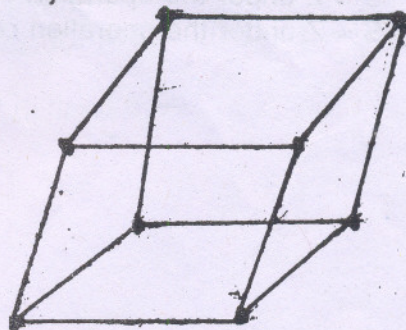


Fig 6.2

7. (a) Consider the square shown in figure-7.1. 6

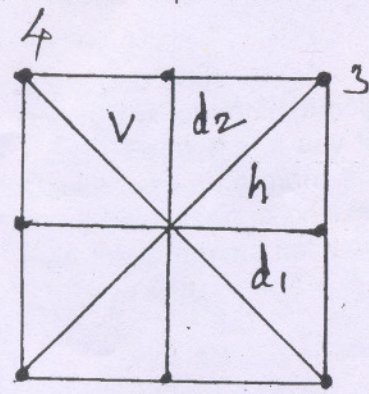


Fig. 7.1

The Symmetries of the square are as follows :—

Rotations  $f_1, f_2, f_3$  and  $f_4$  through  $0^\circ, 90^\circ, 180^\circ$  and  $270^\circ$  respectively,  $f_5$  and  $f_6$  reflections about the lines  $v$  and  $h$  respectively,  $f_7$  and  $f_8$  reflections about the diagonals  $d_1$  and  $d_2$  respectively.

Write the multiplication table of  $D_4$ , the group of Symmetries of the square.

(b) Consider the (2, 5) group encoding function—

$$e : B^2 \rightarrow B^5 \text{ defined by}$$

$$e(00) = 00000 \quad e(10) = 10101$$

$$e(01) = 01110 \quad e(11) = 11011$$

Decode the following works relative to a maximum likelihood decoding function.

- (i) 11110      (ii) 10011      (iii) 10100

(c) Determine whether the graph shown has a Hamilton circuit, a Hamiltonian path but no Hamiltonian circuit or heither. 4

If the graph has a Hamiltonian Circuit, give the circuit.

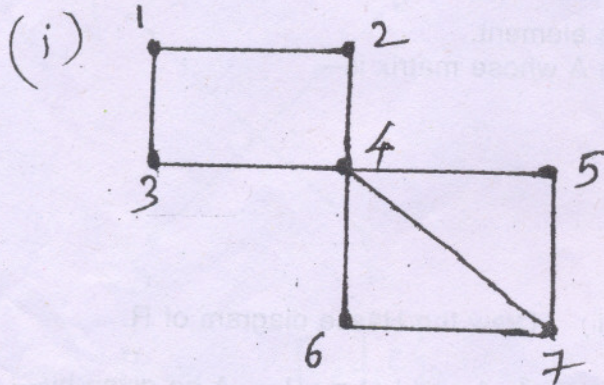


fig 7.2

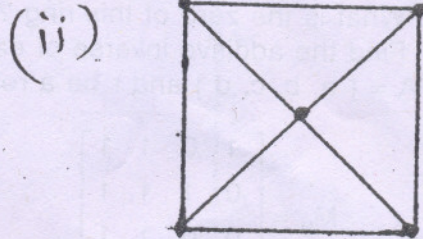


fig 7.3

(d) Determine whether the relation R on the Semigroup S is a Congruence relation. 4

- (i)  $S = Z$  under the operation of ordinary addition ;  $a R b$  if and only if  $a + b$  is even.  
 (ii)  $S = Z$  under the operation of ordinary addition ;  $a R b$  if and only if  $a \equiv b \pmod{3}$ .