Con. 5764–07. (REVISED COURSE) CD–6663

(3 Hours) [Total Marks : 100

N.B. : (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions out of remaining **six** questions.
(3) **Figures** to the **right** indicate **full** marks.

1. (a) Find complex form of Fourier series for $f(x) = e^{ax}$, $(-l, l)$. 5
(b) Define–Analytic function, Harmonic function. If $f(z) = u+iv$ is an analytic function then S. T. u and v are Harmonic functions. 5
(c) If $L\{f(t)\,t\} = \frac{1}{s(s^2+1)}$ find $L\{e^{-t} f(2t)\}$. 5
(d) S. T. the matrix [iA] is Hermitian or Skew-Hermitian according as [A] is Skew-Hermitian or Hermitian. 5

2. (a) If $f(t) = \begin{cases} k & 0 < t < T/2 \\ -k & T/2 < t < T \end{cases}$ 6

and $f(t) = f(t + T)$.

Then S. T. $L\{f(t)\} = \frac{k}{s} \tanh\left(\frac{ST}{4}\right)$.

(b) S. T. $u = e^x \cos y + y^2 - x^2$ is Harmonic function. Find its conjugate Harmonic function. 6
(c) Find Fourier series for 8

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ \sin x & 0 < x < \pi. \end{cases}$$

Hence S.T. $\frac{\pi - 2}{4} = \frac{1}{1\times 3} - \frac{1}{3\times 5} + \frac{1}{5\times 7} \ldots\ldots$

3. (a) Solve using Laplace transform $\frac{d^2y}{dt^2} + 9y = 18t$ given $y(0) = 0$, $y\left(\frac{\pi}{2}\right) = 0$. 6
(b) Find analytic function f(z) in terms of 'z' whose imaginary part is $\frac{y}{x^2 + y^2}$. Hence find its conjugate function. 6
(c) Find for what values of 'k' the given system is consistant. Solve the system for any one value of 'k' $x + y + z = 1$, $x + 2y + 4z = k$, $x + 4y + 10z = k^2$. 8

4. (a) Find A^{-1} by elementary transformation : 6

where $A = \begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$.

(b) Find Bilinear transformation which transforms the points 0, –i, –1 of z-plane into i, 1, 0 of w-plane. Is the Bilinear transformation a parabolic transformation ? 6
(c) Find Fourier series for 8

$$f(x) = \begin{cases} \pi x & 0 \le x < 1 \\ 0 & x = 1 \\ \pi(x-2) & 1 < x \le 2 \end{cases}$$

5. (a) Find nonsingular matrices P and Q such that PAQ is in the normal form 6

$$\text{where, } A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$$

What is the rank of A.

(b) Find :

(i) $L^{-1}\left\{\frac{s}{s^4 + 4a^4}\right\}$ 3

(ii) If $\bar{f}(s) = \log\left[\frac{s^2+1}{s(s+1)}\right]$ find f(t). 3

(c) Find Half range sine series for : 8

$$f(x) = \begin{cases} x & 0 < x < \frac{\pi}{2} \\ \pi - x & \frac{\pi}{2} < x < \pi \end{cases}$$

Hence S. T. $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^4} = \frac{\pi^4}{96}$

6. (a) S. T. a set of functions $\{\sin(2n+1)x\}_{n=0,1,2\ldots\ldots}$ is an orthogonal set of functions over $\left(0, \frac{\pi}{2}\right)$. Hence find orthonormal set of functions. 6

(b) Find Fourier series for $f(x) = |\sin x|$, $-\pi < x < \pi$. 6

(c) Find :

(i) $L\left\{t\int_0^t e^t \sin t\, dt\right\}$ 4

(ii) evaluate $\int_0^{\infty} e^{-2t}\frac{\sin t}{t}\, dt$ 4

7. (a) Find analytic function f(z) if $u - v = (x - y)(x^2 + 4xy + y^2)$. 6

(b) Use convolution theorem to find $L^{-1}\left\{\frac{1}{(s^2+4s+13)^2}\right\}$ 6

(c) S. T. the relation $w = \frac{iz+2}{4z+i}$ transforms the real axis of z-plane into a circle of w-plane. Find the point in z-plane which is mapped on the centre of the circle of w-plane. 8