

(REVISED COURSE)

(3 Hours)

[ Total Marks : 100

- N.B** (1) Question No. 1 is **compulsory**.  
 (2) Attempt any **four** questions out of remaining **six** questions.  
 (3) **Figures** to the **right** indicate **full** marks.

1. (a) Show that  $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$  20

is derogatory and find its minimal polynomial.

(b) If  $A = \begin{bmatrix} \pi & \pi/4 \\ 0 & \pi/2 \end{bmatrix}$  find  $\cos A$ .

(c) Evaluate using Cauchy's Residue theorem.

$I = \int_C \frac{z-1}{(z+1)^2(z-2)} dz$  where C encloses both poles of  $f(z)$

(d) Obtain the estimate of missing figures in the following data :—

x	1	2	3	4	5	6	7	8
y	2	4	8	—	32	—	128	256

2. (a) If  $\lambda$  is an Eigen value of a non-singular matrix A. Prove that  $\frac{|A|}{\lambda}$  is an Eigen value of  $\text{adj } A$ . 6

(b) Find Eigen values and Eigen vectors for the matrix  $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ . 6

(c) Apply Runge Kutta method of fourth order to find and approximate value of  $y$  at  $x = 0.2$  if  $\frac{dy}{dx} = x + y^2$  given that  $y = 1$  when  $x = 0$  in steps of  $h = 0.1$ . 8

3. (a) Find an iterative formula for  $J_N$  where N is a positive number and hence find  $\sqrt{30}$  up to four decimal places. 6

(b) Solve the equations using Gauss Seidal iteration upto two iterations :— 6  
 $6x + y + z = 105$   
 $4x + 8y + 3z = 155$   
 $5x + 4y - 10z = 65$

(c) Use Residue theorem to evaluate : 8

(i)  $\int_0^{2\pi} \frac{\cos 2\theta}{5 + 4 \cos \theta} d\theta$

(ii)  $\int_0^{\infty} \frac{1}{x^2 + 1} dx$

4. (a) Use Lagrange's interpolation formula to find  $f(5)$  from the table :

x	1	2	3	4	7
f(x)	2	4	8	16	129

- (b) Find the characteristic equation of the matrix A given below and hence find the matrix represented by

$$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$$

Where  $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ .

- (c) Show that the matrix A is diagonalisable, also find diagonal form and diagonalising matrix

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

5. (a) Use Newton's forward difference formula to find the number of person's getting wages less than Rs. 15 from :

<b>Wages in Rs.</b>	0—10	10—20	20—30	30—40
<b>No. of Persons</b>	9	30	35	42

- (b) Express  $f(x) = 2x^3 - 3x^2 + 5x - 4$  into factorial polynomial. Also find the function whose first difference is given function.

- (c) Obtain two distinct Laurent's series for  $f(z) = \frac{2z-3}{z^2-4z+3}$  in powers of  $(z-4)$ , indicating the region of convergence.

6. (a) Evaluate  $\int_0^\pi \frac{\sin^2 \theta}{5+4\cos \theta} d\theta$  by dividing  $(0, \pi)$  into  $\sigma$  subintervals by Simpson's rule.  $\frac{3}{8}$ th rule.

- (b) Evaluate  $\int_0^{2+i} (\bar{z})^2 dz$  along the parabola  $2y^2 = x$ .

(c) (i) P.T.  $\mu^2 = 1 + \frac{1}{4}\delta^2$

- (ii) Solve by Gauss Jordan reduction method :

$$x + 2y + z = 8, \quad 2x + 3y + 4z = 20, \quad 4x + 3y + 2z = 16.$$

7. (a) Use Taylor's series method to find the approximate value of

$$y \text{ at } x = 0.2 \text{ given } \frac{dy}{dx} = 1 + y^2, \quad x_0 = 0, \quad y_0 = 0.$$

- (b)  $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$  Find the Eigen value of A. Also find Eigen value of  $4A^{-1}$  and Eigen

vector of  $A^2 - 4I$ .

- (c) State and prove Cauchy's integral formula, use it to evaluate :

$$I = \oint_C \frac{(z+4)^2}{2^4 + 5z^3 + 6z^2} dz \text{ where } C \text{ is } |z| = 1.$$