T.E. Etx | Tele comm' Applied Math Exam 9/1/08

Con. 5644–07. (REVISED COURSE) CD–5988

(3 Hours) [Total Marks : 100

N.B. : (1) Question No. 1 is **compulsory**.

(2) Solve any **four** out of remaining questions.

1. (a) A box contains n tickets numbered 1, 2,----- n. If m tickets are drawn at random from the box. What is the expectation of the sum of the numbers on the tickets drawn ? **20**

(b) Show that $-1 \leq r \leq 1$ where r is the correlation coefficient between two random variables.

(c) Define an equivalence relation. Let A = {1, 2, 3, -----14, 15}. Consider the equivalence relation R defined on A × A by (a, b) R (c, d) if ad = bc. Find the equivalence class of (3, 2).

(d) Using the pigeonhole principle show that if any 11 numbers are chosen from the set {1, 2, 3, -----20} then one of them will be a multiple of another.

(e) The annual rainfall at a certain place is normally distributed with mean 30 mm. If the rainfalls during last 8 years (in mm) are as given. Can we conclude that the average rainfall during last 8 years is less than the normal rainfall ?

2. (a) Explain two applications of χ^2 distribution. To test two methods of instruction, 50 students are selected at random from each of the two groups. At the end of the instruction period, each student is assigned a grade (A, B, C, D, or F) by an evaluating team. The data is recorded as follows : **8**

	Grade					
	A	B	C	D	F	Total
Group I	8	13	16	10	3	50
Group II	4	9	14	16	7	50

Does the data indicate that there is relation between grades and the methods of instruction ?

(b) If f(x) is probability density function of a continuous random variate k_1 mean and variance **6**

$$f(x) = kx^2 \quad 0 \leq x \leq 1$$
$$= (2-x)^2 \quad 1 \leq x \leq 2$$

(c) A = {2, 4, 8, 12, 36} and B = {3, 6, 9, 12, 24} and let $\leq$ be the relation of divisibility. Are the lattices isomorphic ? Draw Hasse diagram. **6**

3. (a) The marks obtained in Mathematics by 1000 students is normally distributed with mean 78% and std deviation 11% : **8**

(i) How many students got marks above 90% ?

(ii) What was the highest mark obtained by lowest 10% of the students ?

(iii) Within what limites did the middle 90% of the students lie ?

(b) Given x = 4y + 5, y = kx + 4 are the lines of regression of x on y and y on x respectively. Show that, 0 < 4k < 1. If $k = \frac{1}{16}$, find the means of two variables and the coefficient of correlation between them. **6**

(c) Let functions f and g be defined by f(x) = 2x + 1 and g(x) = $x^2 - 2$ respectively. Find (i) the composition functions gof, fog and (ii) check if 'f' and 'g' are bijective. **6**

4. (a) What do you mean by a test of significance ? Floppy diskettes manufactured by x and y companies gave the following results. **8**

	x company	y company
No of floppies used	50	50
Mean life in hours	100	120
S D in hours	5	10

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(b) In a precision bombing attack there is 50% chance that any one bomb will strike the target. Two direct hits are required to destory the target completely. How many bombs be dropped to give at least 99% chance of destroying the target ? 6

(c) Calculate the Spearman's rank correlation coefficient for the following data of marks in two subjects Maths and Physics. 6

Maths	80	75	78	93	98	100
Physics	45	65	68	72	71	69

5. (a) Define (i) Lattice (ii) distributive lattice and (iii) complemented lattice. Draw the Hasse diagram of D_{12}, the lattice of divisors of 12 ordered by divisibility. Is D_{12} complemented ? 8

(b) Fit a Poisson distribution to the following data : 6

x	0	1	2	3	4	5	6
f	314	335	204	86	29	9	3

(c) A continuous random variable x has the probability distribution $f(x) = \frac{4}{81} x(9 - x^2)$ when $0 \le x \le 3$ and f(x) = 0, otherswise. Find first four moments about origin and mean. 6

6. (a) Define (i) Ring (ii) Ring with zero divisors. Show that the set s = { 0, 1, 2, 3, 4} is a ring w.r.t. the operation of addition and multiplication modulo 5. 8

(b) Let (G,*) be a group. Prove that G is an abelian group if and only if $(a*b)^2 = a^2*b^2$ where a^2 stands for a*a. 6

(c) The mean value of a random sample of 60 items was found to be 145, with a standard deviation of 40. With 95%. Find limits for the population mean, within 5 of its actual value with 95% or more confidence using the sample mean. 6

7. (a) (i) Let R be a Relation on A. Prove that if R is symmetric, $R = R^{-1}$ and conversly. 8

(ii) If $f:\left\{R \sim \left(\frac{2}{5}\right)\right\} \rightarrow \left\{R \sim \left(\frac{4}{5}\right)\right\}$ is a function defined by $f(x) = \frac{4x+3}{5x-2}$, Prove that f is a bijection and find f^{-1}.

(b) Fit a second degree parabola to the following data by the method of least squares, treating x as the independent variable : 6

x	0·0	0·2	0·4	0·7	0·9	1·0
y	1·016	0·768	0·648	0·401	0·272	0·193

(c) State important features of standard normal distribution. If x_i, i = 1, 2, ---- 50 are independent random variables, each having a Poission distribution with m = 0·03 and $s_n = x_1 + x_2 + \text{----} + x_n$ evaluate $p(s_{50} \ge 3)$. 6