T.E. (Etrx) Sem 6 (Rev.) 18/12/07
Discrete Time Signal Processing

Con. 5480-07. CD-5547

(REVISED COURSE)

(3 Hours) **[Total Marks : 100**

N.B (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions out of the remaining **six questions**.
(3) **Figures** to the **right** indicate **full** marks.
(4) Assume **suitable** data if **necessary**.

1. (a) Determine whether the following signals are periodic : 20
(i) $\cos(0\cdot 01\,\pi n)$ (ii) $\sin 3n$
(b) Test the linearity and time-invariance of the following systems :
(i) $y(n) = \cos[x(n)]$ (ii) $y(n) = x(n) \quad \cos(w_o n)$
(c) Find the energy of the signal :
$$x(n) = \left(\frac{1}{2}\right)^n u(n) + 3^n u(-n-1)$$
(d) Find the Z-transform of the following sequence :
$x(n) = -na^n u(n-1)$
(e) Determine the range of values of 'a' and 'b' for which the LTI system with impulse response as given below is stable :
$$h(n) = \begin{cases} a^n, & n \geq 0 \\ b^n, & n < 0 \end{cases}$$

2. (a) A causal DT system has transfer function H(Z) such that $H(Z) = H_1(Z)\cdot H_2(Z)$.
$H_1(Z)$ has one pole at $Z = \frac{1}{2}$ and one zero at $z = \frac{1}{3}$. $H_2(z)$ has one pole at Z = 0 and zero at $Z = -\frac{1}{2}$.
(i) Find the transfer function of the system 3
(ii) Find the difference equation of the system 2
(iii) Find the response of the system to the input $x(n) = \left(-\frac{1}{2}\right)^n u(n)$. 3
(iv) Draw the pole zero plot of the overall system and hence comment on the stability of the system. 2
(b) State and prove convolution theorem of Z-transform. Using this property, determine the convolution of the following pairs of signals : 10
$$x_1(n) = \left(\frac{1}{2}\right)^n u(n)$$
$$x_2(n) = \left[1 + \left(\frac{1}{2}\right)^n\right] u(n)$$

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3. Consider an LTI system, initially at rest, described by the difference equation :

$$y(n) = \frac{1}{4}y(n-2) + x(n).$$

(a) Determine the impulse response h(n) of the system. **4**

(b) What is the response of the system to the input signal **4**

$$x(n) = \left[\left(\frac{1}{2}\right)^n + \left(\frac{-1}{2}\right)^n\right]u(n).$$

(c) Determine the direct form II, parallel form and cascade form realizations of the system. **6**

(d) Determine and sketch the magnitude response and phase response of the system. **6**

4. (a) Let x(n) = { 1, 2, 3, 4 } **10**

(i) Find x(k) using the DFT equation

(ii) If $x_1(n)$ = { 1, 2, 3, 4, 1, 2, 3, 4 } find $X_1(K)$ using the above result.

(b) Determine the inverse z-transform of $X(z) = \dfrac{1}{1-1\cdot5z^{-1}+0\cdot5z^{-2}}$ if **10**

(i) ROC : $|z| > 1$

(ii) ROC : $|z| < 0{\cdot}5$

(iii) ROC : $0{\cdot}5 < |z| < 1$.

5. (a) Using DITFFT, find DFT $X_1(K)$ of the following sequence : **10**
$x_1(n)$ = { 1, 2, – 1, 2, 4, 2, – 1, 2 }.

(b) If $X_2(k) = X_1(k) - 2$, without performing IDFT, find $x_2(n)$ **5**

(c) If $x_3(n) = x_1(-n)$, without performing FFT, find $x_3(n)$. **5**

6. (a) A second order descrete time system is given by the difference equation : **10**
$y(n) - 0{\cdot}1\,y(n-1) - 0{\cdot}02\,y(n-2) = 2x(n) - x(n-1)$
If the system input is $x(n) = u(n)$ and the initial conditions are $y(-1) = 10$ and $y(-2) = 5$, find :

(i) Zero input response

(ii) Zero state response

(iii) Total response.

(b) A Linear Time-inveriant system is characterized by the system function : **10**

$$H(z) = \frac{3-4z^{-1}}{1-3\cdot5z^{-1}+1\cdot5z^{-2}}$$

Specify ROC of H(z) and determine h(n) for the following conditions :

(i) The system is stable

(ii) The system is causal

(iii) The system is anticausal

(iv) Find the difference equation of the above system.

7. Write notes on any **four** :— **20**

(a) Architecture of DSP processor

(b) Auto-correlation and cross-correlation

(c) Maximum and minimum phase systems

(d) Comparison of computational complexity of DFT and FFT

(e) Linear Phase FIR filter properties.