

Con, 5529-07.

[REVISED COURSE]

CD-5662

(3 Hours)

[Total Marks : 100]

N.B. : Answer any five questions.

1. (a) State the three axioms of probability. 5
- (b) Explain the concept of Joint and conditional probability with one example each. 5
- (c) Let B_1, B_2, \dots, B_n be partitions of an event space $B_i, i = 1, 2, \dots, n$. for the event B that has occurred. Suppose now that an event A occurs. Find expression for $p(B/A)$ in terms of B_i . 10
2. (a) Define and give one example each :- 6
- (i) Probability distribution function of continuous and discrete random variable
- (ii) Probability density function of continuous and discrete random variable.
- (b) Let X be a continuous random variable with p.d.f. $f_x(x) = k \cdot x \cdot (1 - x); 0 \leq x \leq 1$. Find k and determine a number b such that $P(x \leq b) = p(x \geq b)$. Also obtain distribution function. 8
- (c) (i) State important properties of characteristic function. 2
- (ii) Find the probability density function $f_x(x)$ whose characteristic function is given below :- 4
- $$\begin{aligned} \phi_x(w) &= 1 - |w| & ; & |w| \leq 1 \\ &= 0 & ; & |w| \geq 1 \end{aligned}$$
3. (a) If the probability density function of x is 4
- $$f_x(x) = e^{-x} ; x > 0$$
- Find the probability density function of $y = x^3$.
- (b) If x is a continuous random variable with uniform probability density function in $(0, 2\pi)$. Find the probability density function and distribution function of $y = \cos x$. 6
- (c) The joint probability density function of two random variables is given by - 10
- $$\begin{aligned} f_{xy}(x, y) &= 15 e^{-3x-5y} & ; & x > 0, y > 0 \\ &= 0 & ; & \text{else} \end{aligned}$$
- (i) Find the probability that -
- $$1 < x < 2 \text{ and } 0.2 < y < 0.3$$
- (ii) Find the probability that -
- $$x < 2 \text{ and } y > 0.2$$
- (iii) Find the marginal probability distributions of x and y .

4. (a) If x, y are two independent random variables and if $z = x/y$, then prove that probability density function of z is given by – 8

$$f_z(z) = \int_{-\infty}^{\infty} |y| \cdot f_x(yz) \cdot f_y(y) dy$$

- (b) If x and y are two independent random variables with probability density function – 6

$$f_x(x) = e^{-x} ; x > 0 \text{ and}$$

$$f_y(y) = 3 \cdot e^{-3y} ; y > 0$$

Find : $f_z(z)$ if $z = x/y$

- (c) Find the moment generating function (M.G.F) of Poisson distribution and hence find mean and variance. 6

5. (a) Suppose x and y are two random variables. Define covariance and correlation coefficient of x and y . When do we say that x and y are :- 10

- (i) Orthogonal
- (ii) Independent and
- (iii) Uncorrelated ? Are uncorrelated variables independent ?

- (b) If a random process is given by – 10

$$x(t) = A \cos wt$$

Where w is constant and A is random variable with uniform distribution over $(0, 1)$. Find the mean $M_x(t)$, autocorrelation $R_{xx}(t_1, t_2)$ and auto covariance $C_{xx}(t_1, t_2)$ of $x(t)$.

6. (a) What is Random Process ? State four classes of random processes giving one example each. 6

- (b) A Random Process is defined by, $x(t) = \sin(w_0 t + \theta)$ where θ is uniformly distributed in $(0, 2\pi)$ and w_0 is constant. 8

Verify where $x(t)$ is a wide sense stationary process.

- (c) Prove that if the input to a linear time invariant system is w.s.s then the output is also w.s.s. 6

7. (a) Write brief notes on :- 12

- (i) Ergodic Process
- (ii) M / M / 1 Queue
- (iii) Poisson Process.

- (b) (i) State important properties of power spectral density. 2

- (ii) Find the power spectral density of WSS random process whose autocorrelation function is given by – 6

$$R(\tau) = \frac{a^2}{2} \cos b \tau$$