

S.E.-Sem-3 (Rev.)

(REVISED COURSE)

Dec. 2008

Comp.

(3 Hours)

[Total Marks : 100

Applied Mathematics III

9/1/2009

- N.B. : (1) Question No. 1 is compulsory.
 (2) Attempt any four questions out of remaining six questions.
 (3) Use of calculator's (Non-programmable) is allowed.
 (4) Write the sub questions of the main question collectively.

1. (a) Find the z-transform of $f(k) = b^k, K < 0$. 5
 (b) Find L.T. of $2^{3t} + \sqrt{1 + \sin t}$. 5
 (c) Is the following matrix orthogonal? If not, can it be converted into an orthogonal matrix? If yes, how? 5

$$A = \begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$$

- (d) Find the half range cosine series for $f(x) = x, 0 < x < 2$. 5
2. (a) Evaluate $\int_0^{\infty} \operatorname{erf}(2\sqrt{t}) e^{-5t} dt$ 6
 (b) Obtain the Fourier series for— 6

$$f(x) = x + \frac{\pi}{2}, -\pi < x < 0$$

$$= \frac{\pi}{2} - x, 0 < x < \pi$$

- (c) If A and B are two non-singular square matrices of the same order, show that— 8
 $\operatorname{adj.}(AB) = \operatorname{adj.} B \operatorname{adj.} A$

verify the result for $A = \begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix}, B = \begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$

3. (a) Find the L.T. of $\sin \sqrt{t}$. 6
 (b) P.T. every Skew-Hermitian matrix A can be written as $B + ic$ where B is real Skew-symmetric and C is real symmetric matrix. 6
 (c) Find Fourier sine integral representation for $f(x) = \frac{e^{-ax}}{x}$. 8

4. (a) Solve $\frac{d^2y}{dt^2} + \frac{3dy}{dt} + 2y = t \delta(t-1)$ with the conditions $y(0) = 0, y'(0) = 0$ 6

- (b) Show that the rows of the matrix $\begin{bmatrix} 1 & 0 & -5 & 6 \\ 3 & -2 & 1 & 2 \\ 5 & -2 & -9 & 14 \\ 4 & -2 & -4 & 8 \end{bmatrix}$ are linearly dependent and 6

express and row as a linear combination of other rows.

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(c) Find Fourier expansion of $f(x) = \sqrt{1 - \cos x}$ in $(0, \pi)$ Hence deduce that 8

$$\frac{1}{2} = \sum_{n=1}^{\infty} \frac{1}{4n^2 - 1}$$

5. (a) Find $L\{f(t)\}$ where $f(t) = t, 0 < t < 1, f(t) = 0, 1 < t < 2$ and $f(t+2) = f(t)$ for $t > 0$ 6

(b) Find $z \left\{ \sin \left(\frac{k\pi}{4} + a \right) \right\}, k \geq 0.$ 6

(c) Find the Fourier expansion of $f(x) = 2 - 2 < x < 0, f(x) = x, 0 < x < 2.$ 8

6. (a) Prove that $f_1(x) = 1, f_2(x) = x, f_3(x) = \frac{(3x^2 - 1)}{2}$ are orthogonal over $(-1, 1).$ 6

(b) State convolution theorem for z-transform. Hence find $z\{f(k)*g(k)\}$ 6
where $f(k) = U(k)$ and $g(k) = 2^k U(k).$

(c) Find Inverse L.T. of 8

(i) $\frac{s+2}{s^2+4s+7}$

(ii) Using Convolution theorem $\frac{1}{s} \log \left(\frac{s+3}{s+2} \right)$

7. (a) Determine the values of k for which the following equations are consistent. Also 6
solve the system for these values of $k. x + 2y + z = 3, x + y + z = k, 3x + y + 3z = k^2.$

(b) Obtain complex form of Fourier series for $f(x) = \cos h ax + \sin h ax$ in $(-l, l).$ 6

(c) Find the L.T. of - 8

(i) $\int_0^t u^{-1} e^{-u} \sin u \, du$

(ii) $\cos h t \int_0^t e^u \cos h u \, du$