S.E. Sem 3 (Rev.) Comp. Discrete Structures and Graft Theory 22/12/08

430 : TIIndH1208

Con. 5905-08. **(REVISED COURSE)** **RC-8744**

(3 Hours) **[Total Marks : 100**

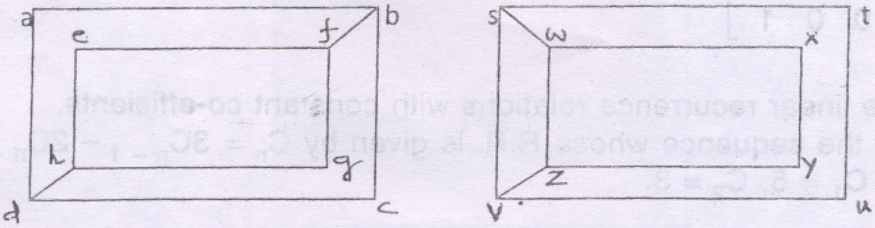
N.B. (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions out of remaining **six** questions.
(3) **Assumptions** made should be **clearly** stated.
(4) **Figures** to the **right** indicate **full** marks.

1. (a) Show that $A \times (B \cap C) = (A \times B) \cap (A \times C)$ 4
(b) Explain with an example the Mutual Inclusion–Exclusion principle for three sets. 4
(c) Find how many integers between 1 and 60 are not divisible by 2, nor by 3 and nor by 5 ? 6
(d) Let A and B two arbitrary sets. 6
Show that $P(A \cap B) = P(A) \cap P(B)$ and also give a counter example.

2. (a) Find how many palindromes of length n can be formed from an alphabet of k letters. 6
(b) Explain with an example different types of quantifiers. 6
Translate the Symbols into English — (i) Ex p(x) and (ii) (x) (~ p(x))
(c) Let A = { a, b, c, d, e } and 4
R = { (a, a), (a, b), (b, c), (c, e), (c, d), (d, e) }
Compute R^2 and R^∞.
(d) Let A = Z , the set of integers and let R be the relation less than. Is R Transitive ? 4

3. (a) Show that if a relation on a set A is transitive and irreflexive, then it is asymmetric. 6
(b) Let A = Z, the set of integers and let R be defined by a R b if and only if $a \leq b$. 6
Is R an equivalence relation ?
(c) Explain the Equivalence Class with a suitable example. 4
(d) Let Z be the set of integers. Define a relation R on Z as a R b iff 6I(a-b). 4
Show that R is an equivalence relation and find ZIR.

4. (a) If 11 numbers are chosen from a set = { 1, 2, ... 20 }. Prove that one of them is multiple of other. 6
(b) If $f : A \to B$ and $g : B \to C$ are both one–one and onto, then 6
$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.
(c) (i) Is every Eulerian graph a Hamiltonian ? 4
(ii) Is every Hamiltonian graph a Eulerian ?
Explain with the necessary graph.
(d) Determine whether graph G and H are isomorphic or not, justify your answer. 4



G H

G H

5. (a) Let Z^+ is a set of positive integers and a relation R defined on Z^+ by a R b iff a|b then prove that R is a partial order relation and (Z^+, |) is a Poset . 6

(b) (i) Determine the Hasse diagram of the relation on A = { 1, 2, 3, 4, 5 } 6

$$M_R = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

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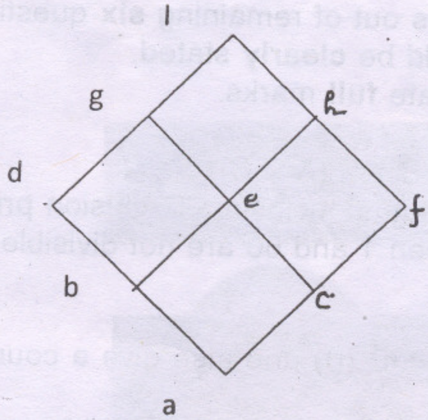
(ii) Draw the Hasse diagram for the Posets A = { 1, 2, 3, 4, 12 }. Under partial order I .

(c) Determine whether the below Hasse diagram represents a lattice. 4



(d) Explain the Extremal Elements of Posets with a suitable example. 4

6. (a) Show that a group G is Abelian if and only if $(ab)^2 = a^2b^2$ for all the elements a and b in G. 6

(b) Let Z_4 i.e. G = { 0, 1, 2, 3 } 6

(i) Prepare its composition table with respect to 'x_4'

(ii) Is it a group ?

(c) Show that R = { a + b $\sqrt{2}$: a, b $\in$ I } is an integral domain. 4

(d) Prove that every field is an integral domain. 4

7. (a) Consider (2, 6) encoding function e : $B^2 \to B^6$ defined as 6

e(00) = 000000
e(01) = 011110
e(10) = 101010
e(11) = 111000

(i) Find the minimum distance.

(ii) How many error can 'e' detect ?

(b) Let m = 2 ,n = 5 and 6

$$H = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

, Determine group code $e_H : B^2 \to B^5$

(c) Explain the linear recurrence relations with constant co-efficients. 4

(d) Determine the sequence whose R.R. is given by $C_n = 3C_{n-1} - 2C_{n-2}$ with initial conditions $C_1 = 5$, $C_2 = 3$. 4