S.E. Sem 3 (Rev.) Etrx. Engineering Mathematics-3 09/1/09

Con. 5728-08. **RC-8726**

(REVISED COURSE)

(3 Hours) **[Total Marks : 100**

N.B. (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions out of **remaining six** questions.
(3) **Figures** to the **right** indicate **full** marks.
(4) Assume suitable data if **required**.

1. (a) Evaluate $\oint_c \left(3x^3 - 8y^2\right) dx + (4y - 6xy)\, dy$ where c is boundary of the region defined by $x = 0,\ y = 0,\ x + y = 1$. 5
   (b) Obtain complex form of the Fourier series for $f(x) = \cosh 4x + \sinh 4x$ in $(-4, 4)$. 5
   (c) If A, B are two non-singular square matrices of same order then show that adj(AB) = adjB adjA. 5
   (d) Define Z-Transform hence find the z-transform of (i) $\dfrac{1}{n(n+1)}$, (ii) $\cos\left(\dfrac{n\pi}{2}\right)$. 5

2. (a) Find the L. T of— (i) $L[(1 + t^{-t})^3]$ 4
   (ii) P.T. $\displaystyle\int_0^\infty e^{-\sqrt{2}t} \sin t \sinh t\,dt = \frac{\pi}{8}$. 4
   (b) Express $f(x) = |x|$ in $(-\pi, \pi)$ as Fourier series hence deduce 6
   $$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}.$$
   (c) Show that the function $f(x) = \sqrt{|xy|}$ is not analytic at origin although Cauchy-Riemann equations are satisfied. 6

3. (a) (i) Show that the following equations $-2x + y + z = a,\ x - 2y + z = b,\ x + y - 2z = c$ have no solution unless $a + b + c = 0$ in which case they have infinitely many solutions. Find these solutions when $a = 1,\ b = 1,\ c = -2$. 4
   (ii) Determine a, b, c and find A^{-1} if $A = \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$ is orthogonal. 4
   (b) Using Fourier cosine integral prove that $e^{-x} \cos x = \dfrac{2}{\pi}\displaystyle\int_0^\infty \frac{\left(\omega^2 + 2\right)}{\left(\omega^2 + 4\right)} \cos \omega x\, dx$ 6
   (c) Find the bilinear transformation which maps the points $z = 1, i - 1$ into the points $w = i, 0, -i$ hence find the image of $|z| < 1$. 6

4. (a) (i) If f(z) is analytic and $|f(x)|$ is constant then prove that f(z) is constant. 4
   (ii) If u (x, y) is harmonic function, then prove that $f(z) = u_x - iu_y$ is anlaytic function. 4
   (b) Find the non-singular matrices P and Q such that PAQ is in normal form. 6
   Also find the rank of A and A^{-1} where $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

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(c) In $(0, \pi)$ Show that — 6

$$x^2 = \frac{2}{\pi}\left[\left(\frac{\pi^2}{1} - \frac{4}{1^3}\right)\sin x - \frac{\pi^2}{2}\sin 2x + \left(\frac{\pi^2}{3} - \frac{4}{3^3}\right)\sin 3x + \ldots\right]$$

5. (a) (i) Using divergence theorem to show that $\oint_s \nabla r^2 \, ds = 6V$ where s is any closed surface enclosing volume V. 4

(ii) Evaluate $\oint_c f \cdot dr$ by Stoke's theorem where $\overline{F} = y^2 i + x^2 j - (x + z) k$ and c is boundary of the triangle with vertics at (0, 0, 0); (1, 0, 0); (1, 1, 0). 4

(b) Solve the equation $(D^2 + 1) y = t$, thegiven that $y(0) = 1$, $y^1(0) = 0$ where $D = \frac{d}{dt}$. 6

(c) Show that the set of function $\sin(2n + 1)x$, n = 0, 1, 2, ... is orthogonal over $\left[0, \frac{\pi}{2}\right]$ hence construct orthonormal sets. 6

6. (a) Find the inverse transformations of

(i) $\cot^{-1}(as)$ 2

(ii) $L^{-1}\left[\frac{e^{-3x}}{(s+4)^3}\right]$ 3

(iii) P.T. $L^{-1}\left[\frac{1}{s}\tan^{-1}\frac{2}{s}\right] = \int_0^t \frac{1}{u}\sin 2u du$ 3

(b) If $F = 3x^2 yz^2 i + x^3 z^2 j + 2x^3 yzk$ then show that $\int_c \overline{f} \cdot d\overline{r}$ is independent of the path of integration, hence evaluate the integral when c is any path joining (0, 0,) to (1, 2, 3). 6

(c) State and prove polar form of Cauchy-Riemann equation hence find the value of p f(z) = where $r^2 \cos 2\theta + i\, r^2 \sin 2\theta$ is analytic. 4+2

7. (a) Find the z-transform of 2+3+3

(i) $e^{3t} \sin 2t$

(ii) Find inverse z-transform $\frac{z^2}{(z-a)(z-b)}$ using convolution theorem

(b) Find the laplace transform of $f(t) = |\sin pt|, t \geq 0$ 6

(c) (i) Using convolution theorem prove that $\int_0^t \sin u \cos(t-u)\, du = \frac{1}{2} t \sin t$ 3

(ii) Find the value of p for which the following matrix A will have (1) rank 1, 3

(2) rank 2, (3) rank 3 where $A = \begin{bmatrix} 3 & p & p \\ p & 3 & p \\ p & p & 3 \end{bmatrix}$.
