S.E. sem 3 (Rev.) EXTC. Applied Maths-3 09/01/09

ws Oct 08 317

Con. 5729–08. **(REVISED COURSE)** **RC–8772**

(3 Hours) **[Total Marks : 100**

N.B. : (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions out of remaining **six** questions.
(3) **Figures** to the **right** indicate **full** marks.

1. (a) Verify if $A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$ is orthogonal. 5

(b) Find the Laplace transform of f(t) where 5

$$f(t) = \begin{cases} \sin \omega t, & 0 < t < \pi/\omega \\ 0, & \pi/\omega < t < 2\pi/\omega \end{cases}$$

(c) Prove that in the interval $0 < x < \pi$, 5

$$\frac{e^{ax} - e^{-ax}}{e^{a\pi} - e^{-a\pi}} = \frac{2}{\pi}\left[\frac{\sin x}{a^2+1} - \frac{2\sin 2x}{a^2+4} + \frac{3\sin 3x}{a^2+9} - \cdots\right].$$

(d) Find the Z-transform of $c^k \cos \alpha k$, $k \geq 0$. 5

2. (a) Obtain the Laplace transform of (i) $t^2 e^t \sin 4t$ (ii) $\int_0^t e^{-2t} \cos^2 t \, dt$. [4+4]

(b) Discuss for what values of λ and μ, the following systems of equations have : 6
(i) no solution (ii) a unique solution and (iii) infinite number of solutions :

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

(c) Obtain the complex form of the Fourier series for $f(x) = \begin{cases} 0, & 0 < x < l \\ a, & l < x < 2l \end{cases}$ 6

3. (a) (i) Show that every square matrix can be uniquely expressed as a sum of Hermitian and Skew-Hermitian matrices. 4

(ii) Check if the following vectors are linearly dependent. If so, find the relation between them : (1, –1, 1), (2, 1, 1), (3, 0, 2). 4

(b) Solve using Laplace transform : 6

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 2y = 5\sin t, \quad y(0) = y^1(0) = 0$$

(c) Find the Fourier series expansion of f(x), a periodic function of period 1, 6

$$\text{where } f(x) = \begin{cases} \frac{1}{2} + x, & \frac{-1}{2} < x \leq 0 \\ \frac{1}{2} - x, & 0 < x \leq \frac{1}{2} \end{cases}.$$

4. (a) Prove that if $L\{f(t)\} = F(s)$, then $L\left\{\frac{1}{t} f(t)\right\} = \int_s^\infty F(s)$. Also evaluate $\int_0^\infty e^{-t} \frac{\sin^2 t}{t} dt$. 8

(b) Find the Fourier cosine integral for $f(x) = \begin{cases} 1-x^2 , & 0 \le x \le 1 \\ 0 , & x > 1 \end{cases}$. 6

Hence evaluate $\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3}\right) \cos \frac{x}{2} dx$.

[TURN OVER

(c) (i) Find the inverse Z-transform of $F(z) = \frac{z+2}{z^2 - 2z + 1}$, $|z| > 1$. 4

(ii) Find the Z-transform of $\{a^{k-1}\}$, $k \geq 0$. 2

5. (a) (i) Find the Laplace transform of $\{t^3 + 2t^2 - 3t + 1\} H(t-1)$. 4

(ii) Find $L^{-1}\left(\frac{1}{\sqrt[3]{s+4}}\right)$. 4

(b) Show that the set of functions $\sin(2n+1)x$, $n = 0, 1, 2, \ldots$ is orthogonal over $\left[0, \frac{\pi}{2}\right]$. Hence construct an orthonormal set of functions. 6

(c) Find the Z-transform of (i) $\left(\frac{1}{2}\right)^{|k|}$ (ii) $\sin(3k+5)$. 6

6. (a) Find non-singular matrices P and Q such that $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$ is in normal form. Also find the rank of A. 8

(b) Find the inverse Z-transform of $F(z) = \frac{1}{(z-3)(z-2)}$, $2 < |z| < 3$. 4

(c) Find the Fourier sine transform of $f(x) = \frac{e^{-ax}}{x}$ and hence evaluate $\int_0^\infty \tan^{-1}\left(\frac{x}{a}\right) \sin x \, dx$. Deduce that $\int_0^\infty \frac{\sin sx}{x} \, dx = \frac{\pi}{2}$. 8

7. (a) (i) Solve by the Gauss-Jordan method : 4

$$3x + 2y - 2z = 4$$
$$x - 2y + 3z = 6$$
$$2x + 3y + 4z = 15$$

(ii) Solve by the Gauss-Seidel method : (Go up to 3 iterations) : 4

$$10x + y + z = 12$$
$$2x + 10y + z = 13$$
$$2x + 2y + 10z = 14$$

(b) (i) Find the Fourier Cosine transform of : 4

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2-x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$$

(ii) State the convolution theorem on Fourier transform. 2

(c) Obtain the Fourier series for the function $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \leq x \leq 0 \\ 1 - \frac{2x}{\pi}, & 0 \leq x \leq \pi \end{cases}$ 6

Deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots$
