

- N.B. : (1) Question No. 1 is **compulsory**.
 (2) Attempt any **four** questions out of remaining **six** questions.
 (3) **Figures** to the **right** indicate **full marks**.

1. (a) Evaluate $\int_c (z - z^2) dz$, where c is the upper half of the circle $|z| = 1$. 5
- (b) Solve the equations by Gauss-Jordan method - 5
 $x + y + z = 9$, $2x - 3y + 4z = 13$, $3x + 4y + 5z = 40$.
- (c) Prove that, Eigen values of a Hermitian Matrix are real. 5
- (d) Evaluate y_4 if $y_0 + y_8 = 122$, $y_1 + y_7 = 24$, $y_2 + y_6 = 5$, $y_3 + y_5 = 20$ if the polynomial is of 7th degree. 5

2. (a) Find the Eigen values and Eigen vectors of the matrix A, 6

$$\text{If } A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$$

- (b) State Cauchy's Residue theorem use it to evaluate - 6

$$\int_c \frac{z^2}{(z-1)^2(z+1)} dz, \text{ where } c \text{ is the circle } |z| = 2.$$

- (c) Using Runge Kutta method of Fourth order to find the approximate value of y at $x = 0.2$ if $\frac{dy}{dx} = x + y^2$, given that $y = 1$ when $x = 0$ in steps of $h = 0.1$. 8

3. (a) Using Lagrange's interpolation formula to find the interpolating polynomial $f(x)$ and $f(3)$ - 6

x	0	1	2	5
f(x)	2	3	12	147

- (b) Test whether the matrix A, 6

$$A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix} \text{ is derogatory and find minimal polynomial}$$

(c) Use Residue theorem to evaluate –

8

(i)
$$\int_0^{2\pi} \frac{1}{5 + 4\cos\theta} d\theta$$

(ii)
$$\int_0^{\infty} \frac{dx}{(x^2 + a^2)^2}, \quad a > 0$$

4. (a) Find a positive root of $xe^x = 1$ lying between 0 and 1 by bisection method. 6
- (b) Examine whether the vectors $x_1 = [3, 1, 1]$, $x_2 = [2, 0, -1]$, $x_3 = [4, 2, 1]$ are linearly independent. 6
- (c) (i) Evaluate $\Delta(x^2 + \sin x)$
- (ii) and $\Delta^2(\cos 3x)$ the interval of differencing being h . 4
- (iii) If $f(1) = 4$, $f(2) = 4$, $f(7) = 5$, $f(8) = 4$, find $f(5)$ using Lagrange's interpolation formula. 4

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5. (a) Solve the following equations by Gauss-Seidel Method

$$27x + 6y - z = 85$$

$$6x + 15y + 2z = 72$$

$$x + y + 54z = 110$$

(b) Verify Cayley - Hamilton theorem for the matrix -

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

and hence find A^{-1} .

(c) Find the Laurents series which represents the function $f(z) = \frac{2}{(z-1)(z-2)}$ when

(i) $|z| < 1$

(ii) $1 < |z| < 2$

(iii) $|z| > 2$

6. (a) Find the value of integral $\int_0^1 \frac{x^2}{1+x^3}$

by using (i) Simpson's $\left(\frac{1}{3}\right)^{\text{rd}}$ rule,

(ii) Trapezoidal rule.

(b) Find the eigen values and eigen vectors for the matrix -

$$\begin{bmatrix} 3 & -1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

(c) State and prove Cauchy's Integral formula and use it to evaluate -

$$\int_c \frac{z+3}{2z^2+3z-2} dz$$

Where c is the circle $|z-i|=2$.

7. (a) Using Taylor's series method obtain the solution of the following equation, correct to four places of decimals

$$\frac{dy}{dx} = 3x + y^2 \text{ with } x_0 = 0$$

$$y_0 = 1 \text{ at } x = 0.1$$

(b) Use Newton's Backward interpolation formula to estimate the profit in the year 1925-

year	1890	1900	1910	1920	1930
Profit in Lakhs	47	65	82	93	102

(c) Evaluate using Residue theorem.

$$\int_c \frac{(z+4)^2}{z^4+5z^3+6z^2} dz$$

where c is $|z|=1$