T.E Sem 5 (Rev.) Comp & I.T.

Applied Maths 5

07/01/09

Con. 5927-08.

RC-6494

(REVISED COURSE)

(3 Hours) **[Total Marks : 100**

N.B.: (1) Question **1** is **compulsory**.
(2) Out of **remaining** questions, attempt any **four** questions.
(3) In all **five** questions to be **attempted**.
(4) Answer to **each** new question to be **started** on a **fresh page**.
(5) **Figures** to the **right** indicate **full** marks.
(6) Use of **statistical table** is **permitted**.

1. (a) A variable takes values 0, 1, 2, ..., n with frequencies proportional to the binomial coefficients 1, n_{c_1}, n_{c_2}, ..., n_{c_n} find the mean and variance of the distributions and show that the variance is half of the mean. **5**

(b) If T is an unbiased estimator of 0, show that T^2 and $\sqrt{T}$ are biased estimators of θ^2 and $\sqrt{\theta}$ respectively. **5**

(c) For two variables x and y with same mean, the two regression equations are $y = ax + b$ and $x = \alpha y + \beta$. Show that $\frac{b}{\beta} = \frac{1-a}{1-\alpha}$. Also find the common mean. **5**

(d) A sportsman's chances of shooting on animal at a distance $r > a$ is $\frac{a^2}{r^2}$. He fires when $r = 2a$ and if he misses, he reloads and fires when $r = 3a, 4a, ...$, if he misses at a distance 'na', the animal escapes. Find the odds against the sportsman. **5**

2. (a) A random variable x has the following probability distribution. **6**

x	−2	−1	0	1	2	3
p(x)	0.1	k	0.2	2k	0.3	3k

(i) Find k.

(ii) Evaluate $P(x < 2)$ and $P(-2 < x < 2)$.

(b) If X is a normal variate with mean μ and standard deviation σ prove that mean deviation about mean is $\frac{4}{5}\sigma$. **6**

(c) Using the method of Lagrangian multipliers solve the N.L.P.P. **8**

Optimize $z = 2x_1^2 + 3x_2^2 + x_3^2$

Subject to $x_1 + x_2 + 2x_3 = 13$

$2x_1 + x_2 + x_3 = 10$

$x_1, x_2, x_3 \geq 0$

3. (a) Obtain the relative maximum or minimum (if any) of the function 6

$$z = x_1^2 + x_2^2 + x_3^2 - 6x_1 - 10x_2 - 14x_3 + 103$$

(b) The marks obtained by students in a degree examination are normally distributed. The mean marks and S.D. of the distribution are 500 and 1000 respectively. If 674 students appeared in the examination and out of these, 550 are to be declared passed. What should be the minimum pass marks ? 6

(c) Using Kuhn - Tucker conditions solve the following N.L.P.P. 8

Maximize $z = 2x_1 + 3x_2 - x_1^2 - x_2^2$

Subject to $x_1 + x_2 \leq 1$

$2x_1 + 3x_2 \leq 6 \qquad x_1, x_2 \geq 0$

4. (a) Calculate the coefficient of correlation between the values x and y below :— 6

x	– 10	– 5	0	5	10
y	5	9	7	11	13

(b) A random sample of 16 values from a normal population showed a mean 41.5 inches and the sum of squares of deviation from this mean equal to 135 square inches. Show that the assumptions of a mean of 43.5 inches for the population is not reasonable. 6

(c) Use penalty (or Big M) method to solve the L.P.P. 8

Maximize $z = 6x_1 + 4x_2$

Subject to $2x_1 + 3x_2 \leq 30$

$3x_1 + 2x_2 \leq 24,$

$x_1 + x_2 \geq 3$

$x_1, x_2 \geq 0$

Is the solution unique ? If not find another solution if the requirement vector $\begin{bmatrix} 30 \\ 24 \\ 3 \end{bmatrix}$

change to $\begin{bmatrix} 24 \\ 30 \\ 3 \end{bmatrix}$ is the solution still optimal ?

5. (a) Use dual simplex method to the solve the L.P.P. 6

Maximize $z = -2x_1 - 2x_2 - 4x_3$

Subject to $2x_1 + 3x_2 + 5x_3 \geq 2$

$3x_1 + x_2 + 7x_3 \leq 3$

$x_1 + 4x_2 + 6x_3 \leq 5$

$x_1, x_2, x_3 \geq 0$

(b) The following tables gives the number of good and bad parts produced by each of three shifts in a factory . 6

	Good Parts	*Bad Parts*
Day Shift	960	40
Evening shift	940	50
Night Shift	950	45

Test whether the production of bad parts is independent of the shifts on which they were produced ?

(c) Prove that Spearman's rank correction coefficient R is given by $R = 1 - \frac{6\Sigma D^2}{N(N^2-1)}$ 8

also find R from the following data.

x	12	17	22	27	32
y	113	119	117	115	121

6. (a) 250 passengers have made reservation for a flight from Mumbai to Nagpur. If the probability that passenger who has reservation will not turn up is 0.016. find the probability that at most 3 passengers will not turn up. 6

(b) 500 articles from a factory are examined and found to be 2% defective. 800 similar articles from second factory are found to have only 1.5% defectives. Can it reasonably concluded that the products of the first factory are inferior to those of second. 6

(c) A company produces two types of goods P and Q that require gold and silver. Each unit of type P requires 2 gms of silver and 1 gm of gold while for Q 1 gm of silver and 2 gm of gold are required. The company has only 90 gms of silver and 80 gms. of gold. Each unit of type P gives a profit of Rs. 400 and each unit of type Q gives a profit of Rs. 500. 8

Formulate the L.P.P. and solve it graphically.

7. (a) Two random samples gave the following information :— 6

Sample	Size	Sample mean	Sum of square of deviation from the mean
1	10	15	90
2	12	14	108

Test whether the samples have been drawn from the same normal population.

(b) The height of men in a city are normally distributed with mean 171 cm and S.D. 7 cm, while the corresponding values for women in the same city are 165 cm and 6 cm respectively. If men and women are chosen from this city, find the probability that the women is taller than the man. 6

(c) (i) Write the dual of the following L.P.P. 2

Maximize $z = x_1 - x_2 + 3x_3$

Subject to constraints

$$x_1 + x_2 + x_3 \leq 10$$
$$2x_1 - x_3 \leq 2$$
$$2x_1 - 2x_2 + 3x_3 \leq 6$$
$$x_1, x_2, x_3 \geq 0$$

(ii) Use simplex method to solve the L.P.P. 6

Maximize $z = 2x_1 + 4x_2 + x_3 + x_4$

Subject to constraints

$$x_1 + 3x_2 + x_4 \leq 4$$
$$2x_1 + x_2 \leq 3$$
$$x_2 + 4x_3 + x_4 \leq 3$$
$$x_1, x_2, x_3 \geq 0$$