T.E. Sem 6 (Rev.)
EXTC. Probability and Random Processes 30/12/08

ws Oct 08 285

Con. 5829–08. **(REVISED COURSE)** **RC–6968**

(3 Hours) **[Total Marks : 100**

N.B. : Answer any **five** questions.

1. (a) State the three axioms of probability. **6**
   (b) Explain the concept of Joint and conditional probability with one example each. **6**
   (c) What is a Random Variable ? Explain continuous and discrete Random Variables with suitable examples. **8**

2. (a) State and prove Bayes' theorem. **8**
   (b) In a factory, four machines A_1, A_2, A_3 and A_4 produce 10%, 25%, 35%, 30% of the items respectively. The percentage of defective items produced by them is 5%, 4%, 3% and 2% respectively. An item selected at random is found to be defective. What is the probability that it was produced by the machine A_2 ? **12**

3. (a) If X is a continuous random variable and Y = a X + b, then prove that – **10**

$$f_y(y) = \frac{1}{|a|} f_x\left(\frac{Y-b}{a}\right)$$

   (b) The Joint density function of two continuous random variables is given by **10**

$$f(x,y) = \begin{cases} xy/8 & 0 < x < 2 \quad 1 < y < 3 \\ 0 & \text{otherwise} \end{cases}$$

   Find : (i) E(x)
   (ii) E(y)
   (iii) E(2x + 3y)

4. A random variable X has the following probability mass function : **20**

X = x	1	2	3	4	5	6
P(X = x)	$^1/_8$	$^3/_8$	$^3/_8$	$^1/_{16}$	$^1/_{32}$	$^1/_{32}$

   (a) Find entropy
   (b) Encode using Shannan Fano and Huffman coding techniques.

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5. (a) Suppose X and Y are two random variables. Define covariance and correlation of X and Y. When do we say that X and Y are **10**
   (i) Orthogonal
   (ii) Independent and
   (iii) Uncorrelated ? Are Uncorrelated variables independent ?

   (b) What is a Random Process ? State four classes of random processes giving one example each. **10**

6. (a) Explain in brief : **10**
   (i) WSS process
   (ii) Poisson process
   (iii) Queueing system.

   (b) The Joint probability function of two random variables X and Y is given by : **10**

$$f(x,y) = \begin{cases} C(x^2 + 2y) & X = 0, 1, 2 \\ 0 \quad \text{otherwise} & Y = 1, 2, 3, 4 \end{cases}$$

   Find : (i) The value of C
   (ii) $P(X = 2, \quad Y = 3)$
   (iii) $P(X \leq 1, \quad Y > 2)$ and
   (iv) Marginal probability functions of X and Y.

7. (a) If X and Y are two random variables with standard deviations σ_x and σ_y and if C_{xy} is the covariance between them, then prove :
   (i) $C_{xy}(x, y) = R_{xy}(x, y) - E(X) \cdot E(Y)$ **4**
   (ii) $|C_{xy}| \leq \sigma_x, \sigma_y$ **4**

   Also deduce that

$$-1 \leq \rho \leq 1.$$ **2**

   (b) Explain power spectral density function. State its important properties and prove any one property. **10**
