

(3 Hours)

[Total Marks : 100]

N.B. : Answer any five questions.

1. (a) State the three axioms of probability. 6  
 (b) Explain the concept of Joint and conditional probability with one example each. 6  
 (c) What is a Random Variable? Explain continuous and discrete Random Variables with suitable examples. 8

2. (a) State and prove Bayes' theorem. 8  
 (b) In a factory, four machines  $A_1$ ,  $A_2$ ,  $A_3$  and  $A_4$  produce 10%, 25%, 35%, 30% of the items respectively. The percentage of defective items produced by them is 5%, 4%, 3% and 2% respectively. An item selected at random is found to be defective. What is the probability that it was produced by the machine  $A_2$ ? 12

3. (a) If  $X$  is a continuous random variable and  $Y = aX + b$ , then prove that - 10

$$f_y(y) = \frac{1}{|a|} f_x\left(\frac{y-b}{a}\right)$$

- (b) The Joint density function of two continuous random variables is given by 10

$$f(x,y) = \begin{cases} xy/8 & 0 < x < 2 \quad 1 < y < 3 \\ 0 & \text{otherwise} \end{cases}$$

Find : (i)  $E(x)$ (ii)  $E(y)$ (iii)  $E(2x + 3y)$ 

4. A random variable  $X$  has the following probability mass function: 20

$X = x$	1	2	3	4	5	6
$P(X = x)$	$1/8$	$3/8$	$3/8$	$1/16$	$1/32$	$1/32$

- (a) Find entropy  
 (b) Encode using Shannon Fano and Huffman coding techniques.

5. (a) Suppose  $X$  and  $Y$  are two random variables. Define covariance and correlation of  $X$  and  $Y$ . When do we say that  $X$  and  $Y$  are
- Orthogonal
  - Independent and
  - Uncorrelated? Are Uncorrelated variables independent?
- (b) What is a Random Process? State four classes of random processes giving one example each.

6. (a) Explain in brief:
- WSS process
  - Poisson process
  - Queueing system.
- (b) The Joint probability function of two random variables  $X$  and  $Y$  is given by:

$$f(x,y) = \begin{cases} C(x^2 + 2y) & X = 0, 1, 2 \\ 0 & \text{otherwise} \end{cases} \quad Y = 1, 2, 3, 4$$

- Find:
- The value of  $C$
  - $P(X = 2, Y = 3)$
  - $P(X \leq 1, Y > 2)$  and
  - Marginal probability functions of  $X$  and  $Y$ .

7. (a) If  $X$  and  $Y$  are two random variables with standard deviations  $\sigma_x$  and  $\sigma_y$  and if  $C_{xy}$  is the covariance between them, then prove:
- $C_{xy}(x, y) = R_{xy}(x, y) - E(X) \cdot E(Y)$
  - $|C_{xy}| \leq \sigma_x \sigma_y$

Also deduce that

$$-1 \leq \rho \leq 1.$$

- (b) Explain power spectral density function. State its important properties and prove any one property.