F. E. Sem-II | Rev | All Branches 7/12/09

(Library) App. mathematics - II.

Con. 3706-09. (REVISED COURSE) 10.30 to 1.30 SP–8445

(3 Hours) [Total Marks : 100

N.B.:(1) Question No. **1** is **compulsory**.

(2) Attempt any **four** questions from the remaining **six** questions.

(3) **Figures** to the **right** indicate **full** marks.

1. (a) Solve $y(x^2y^2 + 2)\,dx + x(2 - 2x^2y^2)\,dy = 0$. 5

(b) Find area of the cardioid $r = a(1 + \cos\theta)$ lying outside the circle $r = \frac{3}{2}a$. 5

(c) Show that $\int_0^2 \left(8 - x^3\right)^{-1/3} dx = \frac{2\pi}{3.\sqrt{3}}$. 5

(d) Prove that $\int_0^1 \frac{x^a - x^b}{\log x} dx = \log\left(\frac{a+1}{b+1}\right)$, using D.U.I.S.rule. 5

2. (a) Solve $\frac{dy}{dx} + \frac{y \log y}{x - \log y} = 0$ 6

(b) Show that $\int_0^\infty x^{m-1} \cos(ax)\,dx = \frac{\Gamma(m)}{a^m} \cos\left(\frac{m\pi}{2}\right)$ 7

(c) Evaluate $\iiint \frac{dxdydz}{\left(x^2+y^2+z^2\right)^{1/2}}$ over the volume bounded by spheres 7

$x^2 + y^2 + z^2 = a^2$ and $x^2 + y^2 + z^2 = b^2$, $b > a > 0$.

3. (a) Evaluate $\iint_R y\,dxdy$ where R is the region bounded by $y^2 = 4x$ and $x^2 = 4y$. 6

(b) Find the length of one arch of the cycloid $x = a(t - \sin t)$, $y = a(1 - \cos t)$ 7

(c) Use Taylor's series method to find $y(1{\cdot}1)$, given $\frac{dy}{dx} = xy^{1/3}$; $y(1) = 1$. 7

Obtain solution of the differential equation directly and compare the answer.

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4. (a) Solve $(D^2 + 3D + 2)y = e^{-2x} + e^x \cos 2x$. 6

(b) Change the order of integration of $\int_0^1 \int_{-\sqrt{2y-y^2}}^{1+\sqrt{1-y^2}} f(x,y)\,dx\,dy$. 7

(c) Using Runge-Kutta method of order four find $y(0\cdot2)$ with $h = 0\cdot1$. 7

Given $\frac{dy}{dx} = \frac{1}{x+y}$; $y(0)=1$.

5. (a) Solve $x^3 \frac{d^3y}{dx^3} + 3x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = x + \log x$ 6

(b) Change to polar co-ordinates and evaluate 7

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x\,dy\,dx}{\sqrt{x^2+y^2}}$$

(c) Find y when $x = 0\cdot05$ by Euler's modified method, taking $h = 0\cdot05$, given that $\frac{dy}{dx} = x^2 + y$; $y(0) = 1$. 7

6. (a) Solve $(xy^3 + y)\,dx + 2(x^2y^2 + x + y^4)\,dy = 0$ 6

(b) Solve $(D^2 + 1)\,y = \operatorname{cosec} x \cot x$ using Variation of Parameter method. 7

(c) Change the order of integration and evaluate $\int_0^\infty \int_0^x x\,e^{\frac{-x^2}{y}}\,dy\,dx$. 7

7. (a) The differential equation for electric charge Q of an electric circuit, consisting of an inductance L, capacitance C and an alternating e.m.f. E sin(nt), applied in series is $L\frac{d^2Q}{dt^2} + \frac{1}{C}Q = E\sin(nt)$. Solve the differential equation to find the charge Q. 6

(b) Find mass of the lamina bounded by $x^2 + 2y - 4 = 0$ and X axis if the density at any point varies as its distance from X axis. 7

(c) Find the volume of the paraboloid $x^2 + y^2 = 4z$ cut off by the plane $z = 4$. 7