

N. B. : (1) Question No. 1 is **compulsory**.

(2) Attempt any **four** questions out of remaining **six** questions. 2.30 to 5.30

(3) Use of **calculator's** (Non-Programmable) is **allowed**.

(4) Write the sub-questions of the **main** question collectively.

(5) **Figures** to the **right** indicates **full** marks.

1. (a) Find the Z-transform of  $f(k) = a^k, k \geq 0$ . 5

(b) Find  $L(\operatorname{erf} \sqrt{t})$  5

(c) Is the following Matrix orthogonal ? If not, can it be converted into an orthogonal matrix ? If yes how ? 5

$$A = \begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$$

(d) Find the half range cosine series for  $f(x) = x, 0 < x < 2$ . 5

2. (a) Evaluate  $\int_0^{\infty} \frac{t^2 \sin 3t}{e^{2t}} dt$ . 6

(b) Obtain the Fourier Series 6

$$f(x) = x + \frac{\pi}{2} \quad -\pi < x < 0$$

$$= \frac{\pi}{2} - x \quad 0 < x < \pi$$

(c) If  $A$  is a square matrix of order  $n$  and  $|A| \neq 0$ , then prove that— 8

(i)  $|A^{-1}| = \frac{1}{|A|}$

(ii)  $\operatorname{adj}(A^{-1}) = (\operatorname{adj} A)^{-1}$

3. (a) Find the Laplace Transformation of  $\frac{\cos \sqrt{t}}{\sqrt{t}}$ . 6

(b) Show that every square matrix can be uniquely expressed as the sum of a Hermitian matrix and Skew-Hermitian matrix. 6

(c) Find Fourier Sine integral representation for  $f(x) = \frac{e^{-ax}}{x}$  8

4. (a) Solve  $\frac{d^2y}{dt^2} + 4y = f(t)$  with conditions  $y(0) = 0, y'(0) = 1$  6  
 and  $f(t) = 1$  when  $0 < t < 1$   
 $f(t) = 0$  when  $t > 1$
- (b) Examine whether the vectors  $X_1 = [3, 1, 1], X_2 = [2, 0, -1], X_3 = [4, 2, 1]$  are 7  
 Linearly independent.
- (c) Examine a Fourier series to represent  $f(x) = x^2$  in  $(0, 2\pi)$  and hence deduce 7  
 that,  $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$
5. (a) Find Laplace transform of  $\text{Sinhat. Sinat.}$  6
- (b) Find the Z - transform of  $\text{Cos}\left(\frac{k\pi}{3} + a\right), k \geq 0.$  6
- (c) Find the Fourier expansion of 8  
 $f(x) = 2$  when  $-2 < x < 0$   
 $f(x) = x$  when  $0 < x < 2$
6. (a) Show that functions  $f_1(x) = 1, f_2(x) = x$  are orthogonal on  $(-1, 1).$  6  
 Determine the constants a and b such that the function  $f_3(x) = -1 + ax + bx^2$   
 Is orthogonal to both  $f_1$  and  $f_2.$
- (b) State Convolution theorem for Z - transform. 6  
 Hence find Z - transform of  $f(k)*g(k)$  where  $f(k) = 4^k U(k)$   
 $g(k) = 5^k U(k)$
- (c) Find Inverse L.T. of 8  
 (i)  $\frac{s^2}{(s^2 - a^2)^2}$   
 (ii)  $\frac{1}{s\sqrt{s+4}}$
7. (a) Test the consistency of the equation and solve them if they are consistent 6  
 $2x - y + z = 9, 3x - y + z = 6, 4x - y + 2z = 7, -x + y - z = 4$
- (b) Obtain the complex form of Fourier series for  $f(x) = \cosh ax$  in  $(-L, L).$  6
- (c) Find the L. T. of 8  
 (i)  $\int_0^t u^{-1} e^{-u} \sin u \, du$   
 (ii)  $\cosh t \int_0^t e^u \cosh u \, du.$