 S.E. (ETRX) Sem III (R)

Con. 5934-09. (REVISED COURSE) SP-7346

Engineering Mathematics-III

(3 Hours) [Total Marks :100

5/1/10

2.30 to 5.30

N.B. (1) Question No. **1** is **compulsory.**
(2) Attempt any **four** questions out of remaining **six** questions.
(3) **Figures** to the **right** indicate **full** marks.

1. (a) State and prove first shifting theorem and hence evaluate $L\{e^{-3t}\cos^2 t\}$. 5

(b) Obtain half range cosine series for $f(x) = x$ in $0 < x < 2$. 5

(c) Show that system of equations – 5

$ax + by + cz = 0,\ bx + cy + az = 0,\ cx + ay + bz = 0$

has non trivial solution if $a + b + c = 0$ or if $a = b = c$.

(d) Find the bilinear transformation which maps the pts $z = \infty, i, o$ onto the pts $w = 0, i, \infty$. 5

2. (a) Find – 9

(i) $L\left\{\frac{e^{-2t}\sin 2t \cosh t}{t}\right\}$ (ii) $L\{t^2 H(t-3)\}$ (iii) $L\left\{t\left(\frac{\sin t}{e^t}\right)^2\right\}$

(b) Solve $(D^2 - 3D + 2)y = 4e^{2t}$ with $y(0) = -3$, $y^1(0) = 5$ using Laplace transform. 5

(c) Determine value of p such that rank of – 6

$$A = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 4 & 4 & -3 & 1 \\ p & 2 & 2 & 2 \\ 9 & 9 & p & 3 \end{bmatrix} \text{ is } 3$$

3. (a) Verify Green's thm $\vec{F} = (x^2 - y^2)\hat{i} + (x+y)\hat{j}$ and c is the traingle with vertices (0,0), (1,1) and (2,1). 8

(b) Find Fourier series of $f(x) = x^2$, $-\pi \le x \le \pi$ and hence prove that — 8

(i) $\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2}$ (ii) $\frac{\pi^2}{12} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$

(iii) $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\cdots$

(c) If $\{f(k)\} = \{2^0, 2^1, 2^2, 2^3, \cdots\}$ find $Z\{f(k)\}$. 4

4. (a) Find :— 9

(i) $L^{-1}\left\{\frac{1}{s(s+4)^2}\right\}$ (ii) $L^{-1}\left\{\frac{3s+1}{(s+1)(s^2+2)}\right\}$

(iii) $L^{-1}\left\{\frac{1}{s}\log\left(\frac{1+2}{s^2}\right)\right\}$

[TURN OVER

(b) Find complex form of Fourier series for $f(x) = \text{Cosh}ax$ in $(-l, l)$ 6

(c) Prove that Every Hermitian matrix A can be written as B + ic, where B is real symmetric and c is real skew symmetric. 5

5. (a) Obtain Fourier expansion of $f(x) = |\text{Cos}x|$ in $(-\pi,\pi)$. 8

(b) Find analytic function $f(z) = u + iv$ such that – 6

$$u - v = \frac{\text{Cos}x + \text{Sin}x - e^{-y}}{2\,\text{Cos}x - e^{-y} - e^{-y}}$$

(c) If $A = \begin{bmatrix} 1 & 2 & 1 \\ a & 0 & 4 \\ 1 & 1 & 1 \end{bmatrix}$ and adj (adj A) = A find a. 6

6. (a) Show that $\vec{F} = (ye^{xy}\,\text{Cos}Z)\,\hat{i} + (xe^{xy}\text{Cos}Z)\,\hat{j} - (e^{xy}\,\text{sin}^2)\,\hat{k}$ is irrotational and find scalar potential of $\vec{F}$ and evaluate $\int_C \vec{F}.d\vec{v}$ along the curve joining the pt (0, 0, 0) and $(-1, 2, \pi)$. 8

(b) Find non singular matrices P and Q such that – 6

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$$ is reduced to normal form. Also find its rank.

(c) Show that $u = \text{Sin}x\,\text{Cosh}y + 2\,\text{Cos}x\text{sin}hy + x^2 - y^2 + 4xy$ satisfies Laplace's equation and find it corresponding analytic function $f(z) = u + iv$. 6

7. (a) Find the inverse Z transform of – 8

$$F(z) = \frac{1}{(z-3)(z-2)}$$

if ROC is –

(i) $|z| < 2$ (ii) $2 < |z| < 3$ (iii) $|z| > 3$

(b) Find Fourier integral representation of – 6

$$f(x) = e^{-|x|} \quad -\infty < x < \infty$$

(c) Find the image of semi infinite strip $x > 0$, $0 < y < 2$ under the transformation $w = iZ + 1$. 6