Con. 5944-09. (REVISED COURSE) SP-7802

S.E. (Comp) Sem IV (R) 6/1/10

Lib

(3 Hours) [Total Marks : 100

Applied Mathematics - IV

2.30 to 5.30

N.B. (1) Question No. **1** is **compulsory**.
(2) Answer any **four** out of remaining **six** questions.
(3) Assume any **suitable** data whenever required and justify the **same**.

1. a) State and prove Cauchy's integral formula. *(5)*

b) Show that following matrices have the same characteristic equation. *(5)*

$$\begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}, \begin{pmatrix} b & c & a \\ c & a & b \\ a & b & c \end{pmatrix}, \begin{pmatrix} c & a & b \\ a & b & c \\ b & c & a \end{pmatrix}$$

c) Find all the basic feasible solution of the equations. *(5)*

$$2x_1 + 6x_2 + 2x_3 + x_4 = 3$$
$$6x_1 + 4x_2 + 4x_3 + 6x_4 = 2$$

d) Find the analytic function if $f(z) = u + iv$ if *(5)*

$$3u + 2v = y^2 - x^2 + 16xy$$

2. a) Show that the matrix $A = \begin{pmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{pmatrix}$ is a diagonisable. *(6)*

Find the diagonal matrix and transforming matrix.

b) Use Simplex method to Maximize $z = 3x_1 + 5x_2$ subject to the constraints *(6)*

$$3x_1 + 2x_2 \le 18$$
$$x_1 \le 4$$
$$x_2 \le 6$$
$$x_1, x_2 \ge 0$$

c) Evaluate (i) $\displaystyle\int_0^{2\pi} \frac{d\theta}{5 + 3\sin\theta}$ (ii) $\displaystyle\int_0^{\infty} \frac{dx}{x^2 + 1}$ *(8)*

3\. **a)** Show that the $A = \begin{pmatrix} 2 & 3 & 4 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{pmatrix}$ is derogatory and find its minimal polynomial. *(6)*

b) Use Big M method to Maximize $z = 3x_1 - x_2$ subject to the constraints *(6)*

$$2x_1 + x_2 \geq 2$$
$$x_1 + 3x_2 \leq 3$$
$$x_2 \leq 4$$
$$x_1, x_2 \geq 0$$

c) Find the bilinear Transformation which maps the points z = 1, i, - 1 onto the points ω= i, 0, -i. Hence find the fixed points of the transformation and the image of $|z| < 1$. *(8)*

4\. **a)** If $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ Find A^{50} *(6)*

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b) Find Laurent's series which represents the function $f(z)=\frac{2}{(z-1)(z-2)}$ when **(6)**

(i) $|z|<1$ (ii) $1<|z|<2$ (iii) $|z|>2$

c) Using the Kuhn Tucker condition solve the NLpp **(8)**

Maximize $z=7x_1^2+5x_2^2+6x_1$ subject to

$$x_1+2x_2\le 10$$
$$x_1-3x_2\le 9$$
$$x_1,x_2\ge 0$$

5. a) If $f(z)=u+iv$ is analytic in R show that **(6)**

$$\left[\frac{\partial}{\partial x}|f(z)|\right]^2+\left[\frac{\partial}{\partial y}|f(z)|\right]^2=|f'(z)|^2$$

b) If $A=\begin{pmatrix}2&2&1\\1&3&1\\1&2&2\end{pmatrix}$ **(6)**

find the characteristic roots and characteristic vectors of A^3+I

c) Using the method of Lagrange's multipliers solve the NLpp **(8)**

optimize $z=x_1^2+x_2^2+x_3^2-10x_1-6x_2-4x_3$

subject to $x_1+x_2+x_3=7$

$x_i\ge 0$

6. a) Apply the principle of duality to solve Lpp **(6)**

Maximize $z=3x_1+4x_2$

subject to the constraints $x_1-x_2\le 1$

$$x_1+x_2\ge 4$$
$$x_1-3x_2\le 3$$
$$x_1,\ x_2\ge 0$$

b) Using residue theorem evaluate $\int_C \frac{e^z}{\left(z^2+\pi^2\right)^2}dz$ where C is $|z|=4$. **(6)**

c) Use dual simplex method to solve the Lpp **(8)**

Maximize $z=-2x_1-2x_2-4x_3$

subject to the constraints $2x_1+3x_2+5x_3\ge 2$

$$3x_1+x_2+7x_3\le 3$$
$$x_1+4x_2+6x_3\le 5$$
$$x_1,\ x_2,\ x_3\ge 0$$

7. a) Obtain the relative maximum or minimum (if any) of the function (6)

$$z = x_1^2 + x_2^2 + x_3^2 - 6x_1 - 10x_2 - 14x_3 + 10_3$$

b) Evaluate $\int_0^{1+i} (x^2 + iy)\,dz$ along (i) $y = x$ (ii) $y = x^2$ (6)

c) Verify Cayley Hamilton Theorem for the matrix (8)

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$ hence find A^{-1}, A^{-2}, A^4.