Con. 6042-09. (REVISED COURSE) SP-7796

S.E. (ETRX) Sem IV (R) 6/01/10

(3 Hours) [Total Marks : 100

Advanced Engineering Mathematics

N.B. : (1) Question No. **1** is **compulsory**.

(2) Attempt any **four** questions out of remaining **six** questions. 2.30 to 5.30 p.m.

1. (a) A box contains 'n' tickets numbered 1, 2, 3, ...n. If m tickets are drawn at random from the box, what is the expectation of the sum of the no. of tickets drawn ? **5**

(b) Prove that the identity element in a group is unique ? **5**

(c) (i) Find the sum and product of the eigen values of $\begin{bmatrix} -2 & -9 & 5 \\ -5 & -10 & 7 \\ -9 & -21 & 14 \end{bmatrix}$ **5**

(ii) Find the eigen values of adjoint of $\begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$

(d) Find Taylor's series expansion of $f(z) = \dfrac{1}{(z-1)(z-3)}$ about the point z = 4. Find region of convergence. **5**

2. (a) Define Random variable with an example. Find k if the following is a p.d.f. **6**

$$f(x) = k\, x\, e^{-4x^2},\ 0 \le x < \infty$$

Also find mean.

(b) Using residue theorem, evaluate $\int_C \dfrac{dz}{4z^2+1}$ where C is |z| = 1. **7**

(c) For x, y ∈, z, x Ry iff 2x + 5y is divisible by 7. Is R an equivalence relation ? Find the equivalence classes. **7**

3. (a) Define level of significance and fiducial limits. A random sample of 625 items from a normal population of unknown mean has mean 10 and standard deviation 1·5. What are 95% and 99% fiducial limits for the population mean ? **6**

(b) If $f(z) = z^3 + iz^2 - 4z - 4i$, evaluate $\int_C \dfrac{f'(z)}{f(z)} dz$ where C encloses zeroes of f(z). **7**

(c) S.T. the set of matrices $M = \begin{bmatrix} a & b \\ -5b & a \end{bmatrix}$ form an integral domain. Is it a field ? **7**

4. (a) Derive Moment generating function for Poisson distribution, hence find mean and variance. **6**

(b) If 2 percent bulbs are known to be defective bulbs, find the probability that in a lot of 300 bulbs, there will be 2 or 3 defective bulbs using (i) Binominal distribution (ii) Poisson distribution. **7**

(c) Are the following functions (i) injective (ii) surjective. 7

Given $f(x) = x^2 - 4x,\ f : R \to R$ and

$$g(x) = \frac{2x-3}{x-1},\ g : R - \{1\} \to R - \{2\}$$

5. (a) Is the following matrix diagonalizable ? Justify your answer. 6

Given $A = \begin{bmatrix} 1 & -2 & 0 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$

(b) If x_1 has mean 5 and variance 5, x_2 has mean -2 and variance 3 and if x_1, x_2 are independent find – 7

(i) $E(x_1 + x_2)$, $V(x_1 + x_2)$

(ii) $E(x_1 - x_2)$, $V(x_1 - x_2)$

(iii) $E(2x_1 + 3x_2 - 5)$, $V(2x_1 + 3x_2 - 5)$

(c) Evaluate $\int_C \frac{z+3}{2z^2+3z-2}\,dz$ where C is the circle with centre at (0,1) and radius 2. 7

6. (a) The marks of 1000 students of a University are normally distributed with mean 70 and standard deviation 5. Estimate the number of students whose marks will be – 6

(i) between 60 and 75

(ii) more than 75

(iii) less than 68.

(b) The average of marks scored by 32 boys is 72 with standard deviation 8 while that of 36 girls is 70 with standard deviation 6. Test at 1% LOS whether the boys perform better than the girls. 7

(c) Draw Hasse diagrams for the following posets, under the relation R 'is divisible by and determine whether they represent lattices. 7

(i) L = { 2, 3, 4, 6, 8, 24, 48 }

(ii) L = { 2, 6, 8, 12, 24 }

7. (a) Fit a binomial distribution to the following data and test goodness of fit. 6

x :	0	1	2	3	4	5	6
f :	5	18	28	12	7	6	4

(b) Verify Cayley Hamilton theorem for $A = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$ and evaluate $2A^4 - 5A^3 - 7A + 6I$. 7

(c) Find the sum of the residues at singular points of $f(x) = \frac{z}{az^2 + bz + c}$ 7