Con. 5267-09. T.E./EXTC/Sem II (R) SP-8618

(REVISED COURSE)

(16) (3 Hours) [Total Marks : 100

Random Signal Analysis 7/12/09 2:30 to 5:30

N.B. (1) Question No. 1 is **compulsory**.
(2) Attempt any **four** questions from Quesntion Nos. **2** to **6**.

1. (a) State total probability theorem and Bay's Theorem. 4
(b) State and prove any **two** properties of— 8
   (i) Density functions
   (ii) Distribution functions.
(c) If $R(\tau)$ is an autocorrelation function then prove that $R(\tau)$ is an even function. 4
(d) Define Markov Chain giving an example. 4

2. (a) A mechanism consists of three paths A, B, C and probabilities of their failure are p, q, r respectively. The mechanism works if there is no failure in any of these parts. Find the probability that— 8
   (i) The mechanism is working
   (ii) The mechanism is not working.
(b) If X, Y are two independent exponentially distributed random variables with same parameter unity, find the probability density function of 12
$$U = X + Y$$
$$V = X/(X + Y).$$

3. (a) A random variable takes values 9, 13, 17 .... (5 + 4n) each with probability 1/n, find mean and variance of X. 8
(b) The joint probability density function of (X, Y) is given by 12
$$f_{XY}(x, y) = K e^{-(X+Y)} \quad 0 < x < y < \infty.$$
Find : (i) K
   (ii) Marginal densities of X and Y
   (iii) Are X, Y iondependent ?

4. (a) State and prove the properties of autocorrelation function and cross correlation function. 10
(b) The power spectral density of random process is given by :— 10
$$S(\omega) = \frac{10\omega^2 + 35}{\left(\omega^2 + 4\right)\left(\omega^2 + 9\right)}$$
Find :—
   (i) Average Power
   (ii) $R(\tau)$ the autocorrelation function.

[TURN OVER

5. (a) If the WSS process X(t) is given by **12**

$$X(t) = 10\cos(100t + \theta)$$

Were θ is uniformly distributed over $(-\pi, \pi)$.

Prove that the X(t) is correlation Ergodic.

(b) Explain Power Spectral Density function. **8**

State its important properties and prove any one property.

6. (a) State and prove Chapman—Kolmogorov Equation. **8**

(b) (i) Define Central Limit Theorem and give its significance **12**

(ii) Define Strong Law of large numbers.

(iii) Describe sequence of random variables.

7. (a) A medical representative visits only three cities A, B, C but he never visits the same city on successive days. If he visits city A today, then he visits city B tomorrow without fail. However, if he visits either city B or C today, then he is twice as likely to visit city A as the other city. **12**

In what proportion does he visit the cities A, B, C in the steady state.

(b) X is continuous random variables with probability density function. **8**

$$f_X(x) = (1/b)e^{-(x-a)/b} \quad ; \; x > a$$

$$= 0 \quad ; \; x < a$$

Find characteristic function $\Phi_X(\omega)$ and hence determine the expected value of X.