

(3 Hours)

[Total Marks : 100

## Signals &amp; Systems

21/12/09

2:30 to 5:30

- N.B. : (1) Question No. 1 is **compulsory**,  
 (2) Answer **five** question in **all**.  
 (3) **All** parts of the same questions must be written in **continuation**.  
 (4) Assume **suitable** data, if **required** and state them **clearly**.

1. Answer any four :-

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(a) A discrete time periodic sequence is given by  $x_p(n) = A \cos \left[ \frac{n\pi}{2} \right]$

- (i) Determine the period of the sequence  
 (ii) Sketch the sequence  $x(n)$  for the variable  $n$  for one period.

(b) Show that  $\int_{-D}^D |x(t)| dt < \alpha$  is a sufficient condition for the existence of the

Fourier transform of  $x(t)$ .

(c) The impulse response of a linear time invariant system is

$$h(n) = \{ 1, 2, 1, -1 \}$$

↑

Determine the response of the system to the input signal

$$x(n) = \{ 1, 2, 3, 1 \}$$

↑

- (d) Derive the initial and final value theorem of  $z$  transform.  
 (e) Determine the direct form - I realization for the following transfer function  
 $H(z) = 1 - 0.5z^{-1} + 0.2z^{-2}$

2. (a) Determine the range of values of the parameter for which the following linear time-invariant systems with impulse response. 10

(i)  $h(n) = a^n u(n)$

(ii)  $h(n) = \begin{cases} a^n & n \geq 0 \\ b^n & n < 0 \end{cases}$  is stable.

(b) Consider the analog signal  $x_a(t) = 3 \cos 100 \pi t$

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- (i) Determine the minimum required sampling rate to avoid aliasing.  
 (ii) Suppose that the signal is sampled at the rate  $F_s = 200$  Hz. What is the discrete time signal obtained after sampling ?  
 (iii) Suppose that the signal is sampled at the rate  $F_s = 75$  Hz. What is the discrete time signal obtained after sampling.

3. (a) A causal system has a difference equation as given below 10  
 $y(n) = 0.5 y(n-1) - 0.25y(n-2) + x(n)$  what is the ROC of the transfer function of the system.

(b) (i) Find the condition for an LTI DTS described as 10

$$y(k+1) - \alpha y(k) = \alpha r(k)$$

to be BIBO stable.

(ii) Check the BIBO stability of T.F  $H(z) = \frac{z-2}{z(z-0.8)}$

[TURN OVER

4. (a) Find the Fourier transform of the unit impulse train function  $\delta_T(t)$ , where  $\delta_T(t)$  is defined by

$$\delta_T(t) = \dots + \delta(t + 2T) + \delta(t + T) + \delta(t - T) + \delta(t - 2T) + \dots \} = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

- (b) Evaluate the following convolutions -

(i)  $\delta(t) * \delta(t)$

(ii)  $x(t) * \delta(t - t_0)$

5. (a) Find the z transform of -

(i)  $x(n) = (-1)^n \cos \frac{\pi}{3} n \quad u(n)$

(ii)  $x(n) = (n + 1)a^n$ .

- (b) Determine a cascade realization for the following transfer function

$$H(z) = \frac{0.7(z^2 - 0.36)}{z^2 + 0.1z - 0.72}$$

6. (a) Find the Fourier series for the function  $x(t)$  defined by

$$x(t) = \begin{cases} 0 & -\frac{T}{2} < t < 0 \\ A \sin \omega_0 t & 0 < t < \frac{T}{2} \end{cases}$$

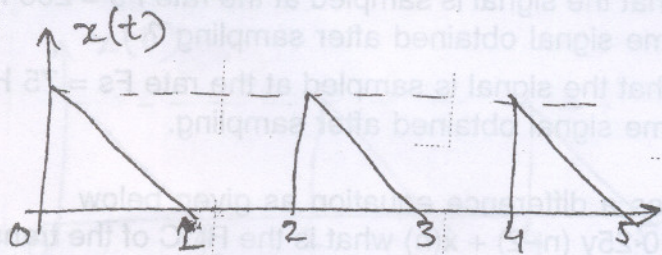
and  $x(t + T) = x(t)$ ,  $\omega_0 = \frac{2\pi}{T}$

- (b) A discrete time system is described by the difference equation,

$$y(k - 3) + 2y(k - 2) + 3y(k - 1) + y(k) = 2u(k).$$

Draw the block diagram representing the system. Generate the state variable description in terms of appropriate A, B, C & D matrices.

7. (a) Show that if  $x_e(n)$  is an even signal and  $x_o(n)$  is an odd signal, then  $x_e(n) \cdot x_o(n)$  is an odd signal. 20
- (b) Find the Laplace transform of the signal shown below



- (c) Find the state transition matrix for

$$A = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}$$

- (d) Obtain the inverse z transform of a rational function

$$f(z) = \frac{z^2}{z^3 - 1.7z^2 + 0.8z + 0.1}$$