Con. 6046–09. T.E.(EXTC) Sem VI 7/01/10

SP–8108

(3 Hours) [Total Marks : 100

(16) Probability & Random Processes

N.B. : Answer any **five** questions. 2.30 to 5.30 PM

1. (a) If A and B are independent events, then prove that $P(A \cap \overline{B}) = P(A) \cdot P(\overline{B})$ and if A and B are exclusive events, prove that $P(A \cap \overline{B}) = P(A)$. 5

   (b) State and explain with an example – 5
   (i) Conditional Probability
   (ii) Baye's Theorem.

   (c) For a certain binary communication channel, the probability that a transmitted '0' is received as '0' is 0·9 while the probability that a transmitted '1' is received as '1' is 0·8. If the probability of transmitting '0' is 0·45, find the probability that – 10
   (i) a 1 is received
   (ii) a 0 is received
   (iii) a 1 was transmitted given that 1 was received
   (iv) a 0 was transmitted given that 0 was received.

2. (a) Define probability density function. State and prove properties of probability density function. 5

   (b) Define and give example of expectation of continuous and discrete random variable. 5

   (c) Consider the random variable x with p.d.f. $f_x(x)$ given by – 10

   $$f_x(x) = A(1 + x) \quad ; \quad -1 < x \leq 0$$
   $$= A(1 - x) \quad ; \quad 0 < x < 1$$
   $$= 0 \quad ; \quad \text{else}$$

   (i) Find A and Plot $f_x(x)$
   (ii) Find $f_x(x)$
   (iii) Find point b such that –

   $$P[x > b] = \frac{1}{2} P[x \leq b]$$

3. (a) If X is Poisson distributed random variable, find moment generating function (M. G. F.) and characteristic function. 8

   (b) Define the characteristic function $\phi_x(w)$ of random variable X. 12

   Show that $\phi_x(w)$ can be expressed as $\phi_x(w) = \sum_{n=0}^{\infty} m_n \frac{j^n w^n}{n!}$

   Where $m_n = \frac{1}{j^n} \left[\frac{d^n}{dw^n} \phi_x(w) \right]_{w=0}$ is the n^{th} order moment of r.v.x.

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4. (a) If X is continuous random variable and Y = g(x) is strictly monotonic function of X, then prove that $f_y(y) = f_x(x).\left|\frac{dx}{dy}\right|$. 5

(b) If probability density function of X is $f_x(x) = e^{-x}$; $x > 0$ 5
Find the probability density function of $Y = x^3$.

(c) The joint probability density function of two random variables is given by 10

$$f_{x\,y}(x, y) = 15\, e^{-3x-5y} \quad ; \quad x > 0,\ y > 0$$
$$= 0 \quad ; \quad \text{else}$$

(i) Find the probability that –
$1 < x < 2$ and $0{\cdot}2 < y < 0{\cdot}3$

(ii) Find the probability that –
$x < 2$ and $y > 0{\cdot}2$

(iii) Find the marginal probability distributions of x and y.

5. (a) If two random variables are independent and if z = x + y, then prove that probability density function of their sum is given by convolution of their density functions. 10

(b) If x and y are two independant exponential random variables with probability density functions : 10

$$f_x(x) = 2\, e^{-2x} \quad ; \quad x \geq 0$$
$$= 0 \quad ; \quad x < 0$$
$$\text{and } f_y(y) = 3\, e^{-3y} \quad ; \quad y \geq 0$$
$$= 0 \quad ; \quad y < 0$$

Find the probability density function of z = x + y.

6. (a) Consider the process x(t) = A cos wt + B sin wt, where A and B are uncorrelated random variables with mean zero and variance 1 and w is positive constant. Show that the process x(t) is covariance stationary. 8

(b) Define : 6

(i) Mean
(ii) Auto correlation and
(iii) Auto covariance of random process.

(c) Prove that if input to LTI system is w.s.s. then the output is also w.s.s. 6

7. (a) Explain power spectral density function. State its important properties and prove any one property. 10

(b) Explain in brief : 10

(i) M / M / 1 Queue
(ii) Poisson process.
