S.E. Electronic / Sem III

P4–Con No–11

Con. 5531–10. Engineering Math III GT–6216

(3 Hours) [Total Marks : 100

N.B. : (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions out of remaining **six** questions.
(3) **Figures** to **right** indicates the **full** marks.

1. (a) Find $L\left[t\sqrt{1+\sin t}\right]$. 05

(b) Find the Z-transform of Discrete Unit Step function,

$U(k) = 1, \quad k \geq 0$
$= 0, \quad k < 0$

Draw the graph of the given function and write the ROC. 05

(c) Determine $a.b, c$ & d so that the function $f(z) = \left(x^2 + axy + by^2\right) + i\left(cx^2 + dxy + y^2\right)$ is analytic. 05

(d) If $f(x) = C_1\phi_1(x) + C_2\phi_2(x) + C_3\phi_3(x)$, where C_1, C_2, C_3 are constants and ϕ_1, ϕ_2, ϕ_3 are orthonormal functions on (a, b), show that $\int_a^b [f(x)]^2 \, dx = C_1^2 + C_2^2 + C_3^2$. 05

2. (a) Derive Cauchy-Riemann equations in polar coordinates and find p if $f(z) = r^2 \cos 2\theta + ir^2 \sin p\theta$ is analytic. 06

(b) Show that $\{\cos x, \cos 2x, \cos 3x, \ldots\}$ is a set of orthogonal functions over an interval $(-\pi, \pi)$. Hence construct the corresponding set of orthonormal functions. 06

(c) Verify Greens's theorem for $\overline{F} = x^2 i - xy j$ and C is the triangle with vertices $A(0, 2)$, $B(2, 0)$, $C(4, 2)$. 08

3. (a) Find the Laplace transform of each of the following:- 06

(i) $\int_0^t u \cos^2 u \, du$ (ii) $te^{3t} erf\sqrt{t}$

(b) Find complex form of Fourier series for $f(x) = e^{-x}$, $-\pi < x < \pi$. 06

(c) Find non-singular matrices P and Q such that PAQ is in normal form. 08
Hence find (i) rank A (ii) A^{-1},

where $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

4. (a) Solve the system of equations $x + 2y - z = 1$, $x + y + 2z = 9$, $2x + y - z = 2$. 06

(b) Find the inverse Laplace transform of the following:- 06

(i) $\frac{e^{4-3s}}{(s+4)^{5/2}}$ (ii) $\frac{8e^{-3s}}{s^2+4}$.

(c) If $f(x)$ is periodic with period 2π and $f(x) = -\pi, \quad -\pi < x < 0$
$= x, \quad 0 < x < \pi,$ 0

obtain Fourier Series of $f(x)$ in $(-\pi, \pi)$. Deduce – $\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} = \frac{\pi^2}{8}$.

5. (a) Using Convolution theorem, find inverse Laplace transform of $\frac{1}{s^2(s+1)^2}$. 0

(b) Find the analytic function and its imaginary part if real part is $\frac{\sin 2x}{\cosh 2y + \cos 2x}$. 0

(c) Prove that $\overline{F} = (y^2 \cos x + z^3)\, i + (2y \sin x - 4)\, j + (3xz^2 + 2)\, k$ is a conservative field. Find (i) scalar potential for $\overline{F}$ (ii) the work done in moving an object in this field from (0, 1, −1) to ((π/2, −1, 2).

6. (a) Using Laplace transformation, solve the following equation

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} + 8y = 1 \quad , \quad y = 0 \text{ and } \frac{dy}{dt} = 0 \text{ at } t = 0.$$

(b) Find orthogonal trajectories of u=constant where $u = x^2 - y^2 - 2xy + 2x - 3y$.

(c) Find the inverse Z-transform of –

$$F(z) = \frac{1}{(z-3)(z-2)}$$

if ROC is (i) $|z| < 2$ (ii) $2 < |z| < 3$ (iii) $|z| > 3$.

7. (a) Evaluate $\int_0^{\infty} t\, e^{-3t} \sin t\, dt$ by using Laplace transform.

(b). Find the bilinear transformation which maps the points $z = 1,\ i,\ -1$ into the points $w = i,\ 0,\ -i$. Also determine the image of $|z| < 1$ under this transformation.

(c) Express the function –

$$f(x) = 1, \quad |x| \le 1$$
$$= 0, \quad |x| > 1$$

as a Fourier integeral and hence evaluate – $\int_0^{\infty} \frac{\sin w \cos wx}{w}\, dw$.

SE/ETRx/Sem III
Sub = BEC

Con. 6341-10. GT-6222

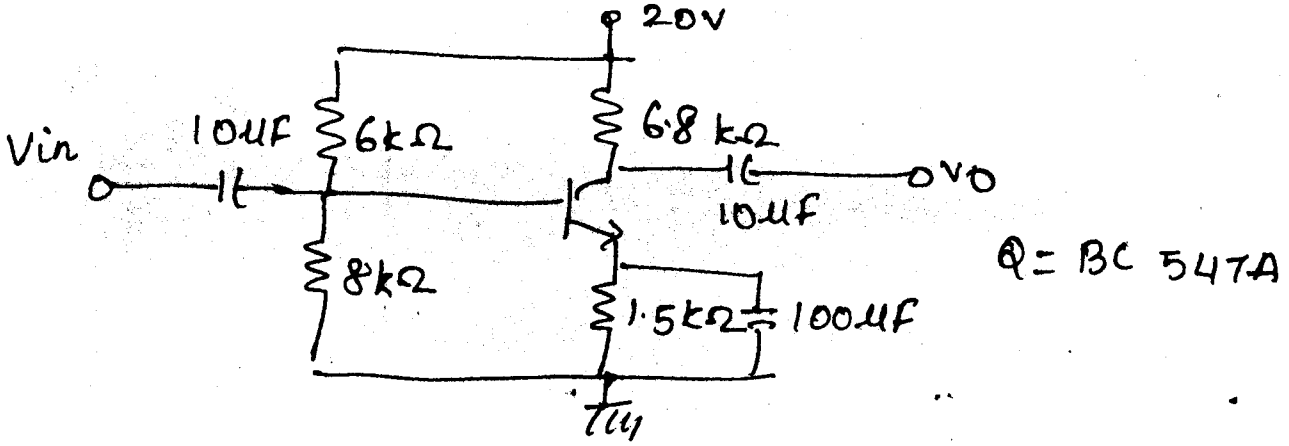
Basic Electronic Circuits (3 Hours) [Total Marks : 100

N.B. : (1) Question No. **1** is **compulsory**.
(2) Answer any **four** out of remaining **six** questions.
(3) Assume any **suitable** data wherever **required.**

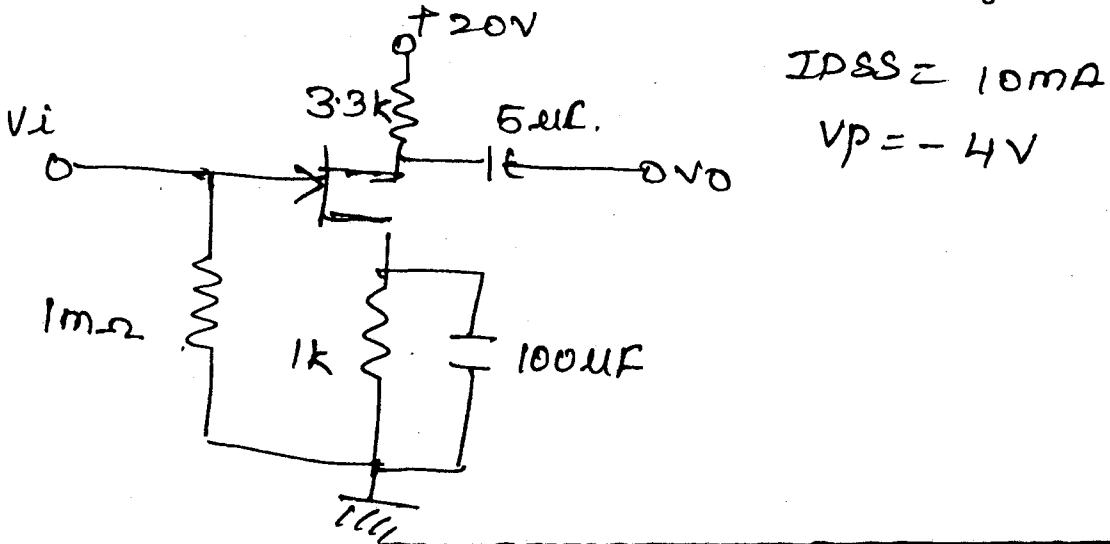
1. (a) What is Maxium reverse voltage across a diode in— **5**
   (i) HWR
   (ii) FWR with center tapped transformer
   (iii) Bridge type rectifier ?
   (b) Campare BJT and FET. **5**
   (c) Draw and explain Negative clamper circuit. **5**
   (d) Explain Thermal stabalization in BJT. **5**

2. (a) Design a single stage RC Coupled CE Amplifier to meet the following requirements— **15**
   $zi \geq 3\ k\Omega$ Vo peak = 2 v, $|AV| \geq 100$, $S \leq 8$.
   (b) Explain the Construction and Working of VMOS. **5**

3. (a) Sketch and Explain the working, Construction and Characteristic of JFET. **10**
   (b) For the following circuit determine zi, zo, Av and Ai :— **10**



4. (a) Design for a fullwave rectifier, an L type LC filter which gives a dc output voltage of 10 V at a load current of 100 MA. The allowable ripple factor is 0·02. **10**
   (b) Determine the equation for AV, Zi and Zo and determine AV, Zi and Zo for a given network. **10**



5. (a) Explain different biasing technique for EMOSFET. 10
   (b) Expalin with neat diagram Zener as regulator. 10

6. (a) Explain hybrid Model of BJT. 10
   (b) Draw the voltage doubler circuit and explain its working. 10

7. Write short notes on the following :— 20
   (a) Photodiode and photo-voltaic cell
   (b) Transistor as a constant current source
   (c) LC filter
   (d) E and D MOSFET.

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DBEC DATA SHEET

Transistor type	Pdmax @ 25°C Watts	Icmax @ 25°C Amps	$V_{CE}^{(sat)}$ volts d.c.	V_{CBO} volts d.c.	V_{CEO} (Sus) volts d.c.	V_{CER} (Sus) volts d.c.	V_{CEX} volts d.c.	V_{EBO} volts d.c.	T_j max °C	D.C. current gain min	typ.	max.	Small Signal min.	typ.	h_{fe} max.	V_{BE} max.	θ_{fc} °C/W	Derate above 25°C W/°C
2N 3055	115·5	15·0	1·1	100	60	70	90	7	200	20	50	70	15	50	120	1·8	1·5	0·7
ECN 055	50·0	5·0	1·0	60	50	55	60	5	200	25	50	100	25	75	125	1·5	3·5	0·4
ECN 149	30·0	4·0	1·0	50	40	—	—	8	150	30	50	110	33	60	115	1·2	4·0	0·3
ECN 100	5·0	0·7	0·6	70	60	65	—	6	200	50	90	280	50	90	280	0·9	35	0·05
BC147A	0·25	0·1	0·25	50	45	50	—	6	125	115	180	220	125	220	260	0·9	—	—
2N 525(PNP)	0·225	0·5	0·25	85	30	—	—	—	100	35	—	65	—	45	—	—	—	—
BC147B	0·25	0·1	0·25	50	45	50	—	6	125	200	290	450	240	330	500	0·9	—	—

Transistor type	hie	hoe	hre	θja
BC 147A	2·7 K Ω	18μ ℧	$1·5 \times 10^{-4}$	0·4°C/mw
2N 525 (PNP)	1·4 K Ω	25μ ℧	$3·2 \times 10^{-4}$	—
BC 147B	4·5 K Ω	30μ ℧	2×10^{-4}	0·4°C/mw
ECN 100	50 Ω	—	——	——
ECN 149	15 Ω	—	——	——
ECN 055	12 Ω	—	——	——
2N 3055	6 Ω	—	——	——

BFW 11—JFET MUTUAL CHARACTERISTICS

–Vgs volts	0·0	0·2	0·4	0·6	0·8	1·0	1·2	1·6	2·0	2·4	2·5	3·0	3·5	4·0
Ids max. mA	10	9·0	8·3	7·6	6·8	6·1	5·4	4·2	3·1	2·2	2·0	1·1	0·5	0·0
Ids typ. mA	7·0	6·0	5·4	4·6	4·0	3·3	2·7	1·7	0·8	0·2	0·0	0·0	0·0	0·0
Ids min. mA	4·0	3·0	2·2	1·6	1·0	0·5	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0

N-Channel *JFET*

Type	V_{DS} max. Volts	V_{DG} max. Volts	V_{GS} max. Volts	P_d max. @25°C	T_j max.	I_{DSS}	g_{mo} (typical)	$-V_p$ Volts	r_d	Derate above 25°C	θ_{ja}
2N3822	50	50	50	300 mW	175°C	2 mA	3000 μ ℧	6	50 KΩ	2 mW/°C	0·59°C/mW
BFW 11 (typical)	30	30	30	300 mW	200°C	7 mA	5600 μ ℧	2·5	50 KΩ	——	0·59° C/mW

S. E. / ETRx / Sem III

Electrical - Net Analy & Synthesis

Con. 5587-10. **GT-6225**

(3 Hours) **[Total Marks : 100**

N.B. : (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions out of the remaining **six** questions.
(3) Assume any **suitable data** if **required**.
(4) **Figures** to the **right** indicate **full** marks.

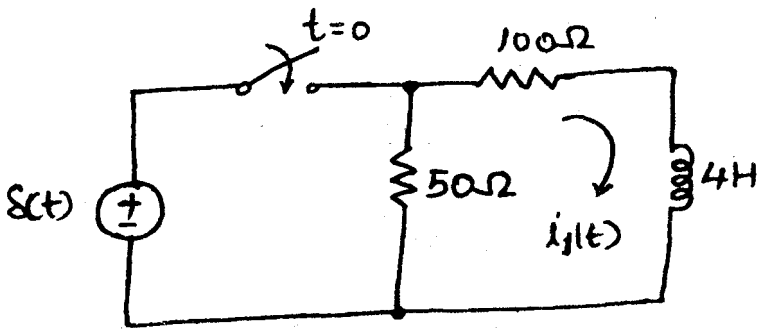
1. (a) Derive the expression for Z-parameters in terms of Y parameters. **5**
The impedance parameters of a two port network are $Z_{11} = 6\ \Omega$, $Z_{22} = 4\ \Omega$, $Z_{12} = Z_{21} = 3\ \Omega$. Compute the Y parameters.

(b) The incidence matrix is given below :— **5**

$$\Lambda = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 1 \\ 1 & 0 & 0 & 0 & -1 & -1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Draw the oriented graph.

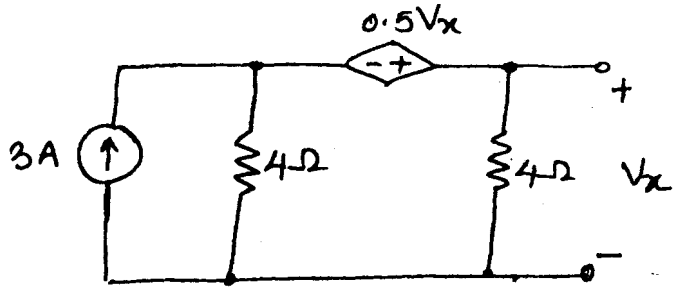
(c) For the network shown, the switch is closed at t = 0. Find the current $i_1(t)$ for t > 0. **5**



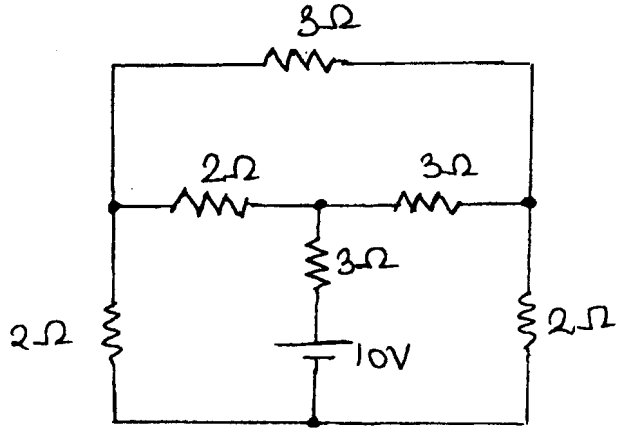
(d) Determine whether the following function is positive real. **5**

$$F(s) = \frac{s^4 + 3s^3 + s^2 + s + 1}{s^3 + s^2 + s + 1}$$

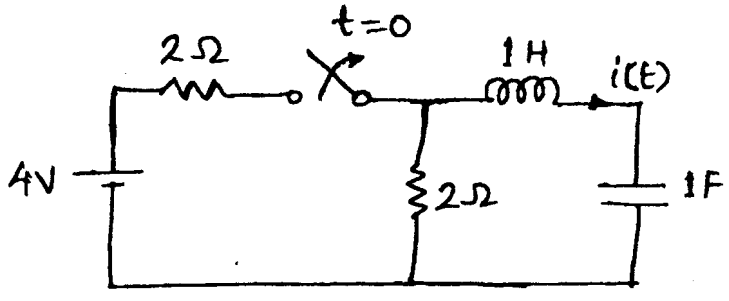
2. (a) Find the Thevenin's and Norton's equivalent network for the network given below. **10**



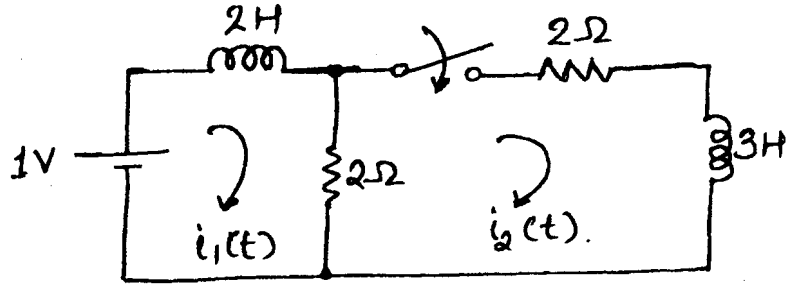
(b) For the given network, draw the oriented graph and obtain incidence matrix, fundamental cutset matrix, and fundamental tie-set matrix. How many trees are possible for the graph ? **10**



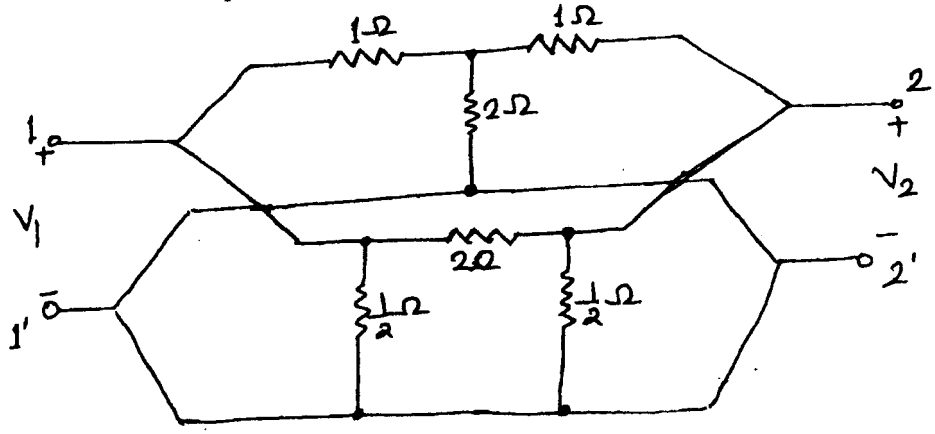
3. (a) The switch in the given network is opened at t = 0. Find i(t) for t > 0. **10**



(b) In the network shown below, the switch is closed at t = 0, the steady-state having been reached before t = 0. Determine the current through inductor of 3H using Laplace transform. **10**

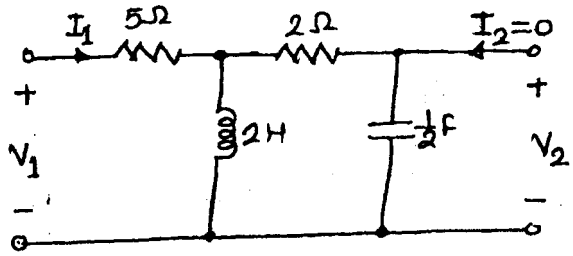


4. (a) Determine Y-parameters of the network. **10**

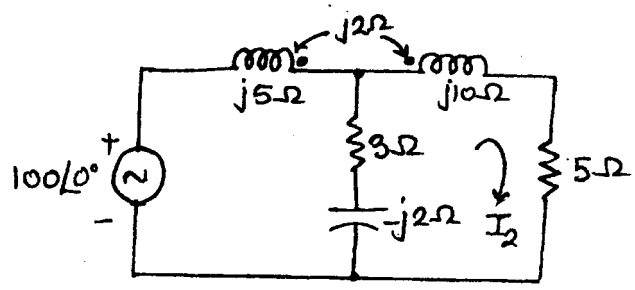


(b) Determine the driving point impedance $\frac{V_1}{I_1}$, transfer impedance $\frac{V_2}{I_1}$ and **10**

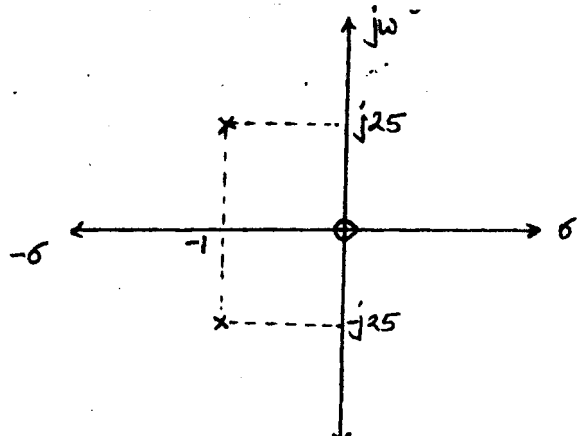
voltage transfer ratio $\frac{V_2}{V_1}$ for the given network.



5. (a) Find I_2 using mesh analysis. **8**

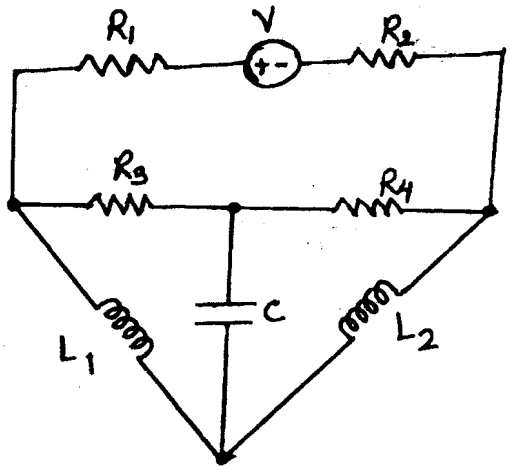


(b) A series RLC circuit has scale factor 5 for its driving point admittance, pole-zero diagram is given in **figure** below. Find the values of R, L and C. **8**



[TURN OVER

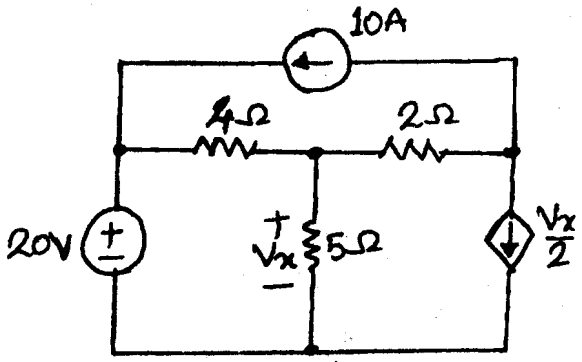
(c) Draw the dual of the network. 4



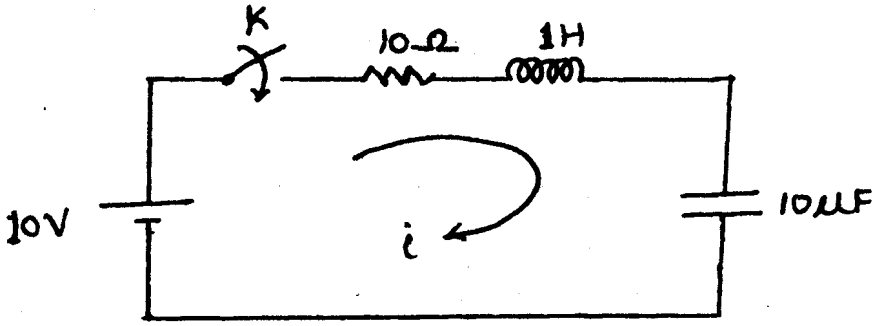
6. Realize the given impedance function using all four forms. 20

$$Z(s) = \frac{(s+1)(s+3)}{s(s+2)}.$$

7. (a) Find V_x using superposition theorem. 10



(b) In the network shown in **figure** below, switch K is closed at t = 0. Assuming all initial conditions zero find i, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t = 0^+$. 10



S.E. Electronics / Sem III

Digital System Design - I

ws Oct Sacn- 10 70

Con. 5999–10. **GT–6213**

(3 Hours) **[Total Marks : 100**

N.B. : Question No. **1** is **compulsory**. Solve any **four** from rest **six**.

1. Answer the following questions 20

(a) Define the following in your own words:
- i) Fan-out of a gate
- ii) Noise margin
- iii) Propagation delay
- iv) Digital multiplexing
- v) Setup and hold times for a flip-flop

(b) What are the advantages of an edge triggered flip-flop over a level triggered device. Explain with example.

(c) Convert BCD numbers to excess-3 codes.

(d) Convert to canonical forms:
i) F(X,Y,Z) = X.Y' + X.Z
ii)F(A,B,C) = (A+B').(A'+C')

2. (a) Obtain minimum SOP expressions for the following using K-maps, implement using universal gates.

i) $F(W,X,Y,Z) = \Sigma m(2,3,6,7,8,9,12,13) + d(4,10,14)$ 3

ii) $F(W,X,Y,Z) = \Sigma m(0,3,4,5,6,7,11,12,13,14,15)$ 3

iii) $F(V,W,X,Y,Z) = \Pi M(2,3,6,7,12,13,14,15,18,19,22,23,25,27,28,29)$ 4

(b)Show how IC 7483 , 4-bit adder, can be cascaded to add two 16-bit numbers. Explain with example. 10

3. (a) Simplify the following 5 variable Boolean expression using Quinn- McClusky tabulation algorithm: 10

$F = \Sigma m (1,3,4,5,6,9,11,12,13,14,20,21,22,29,30,31) + d(7,23,28)$

(b) Design a 4-bit Johnson (twisted ring) counter using flip-flops of your choice. Explain using waveforms. 10

4. (a) Design an clocked MN flip-flop using JK flip-flop. The function table of MN flip-flop is as follows: 10

M	N	Q_{n+1}
0	0	Q_n'
0	1	0
1	0	1
1	1	Q_n

(b) Explain and draw MOD – 12 asynchronous counter using T- FF. Draw output waveforms and show where glitches occur. 10

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5. (a)Show circuit implementations with minimum no. of gates in each of the following forms for the function F= X′ .Y + X.Y′ 10
   i) AND / OR
   ii) NAND / NAND
   iii) NOR / NOR
   iv) NAND / AND

   (b)Determine whether any static 0 or static 1 hazards exist in the following Boolean equation. Identify where the hazards are and what must be done to avoid them. 10
   $F(A,B,C,D) = \Sigma m(5,7,8,9,10,11,13,15)$

6. (a) Sketch the diagram of a TTL Nand gate. Explain how the gate output can be put into high impedance state. 10

   (b) Draw and explain a 16 - bit even parity checker using IC 74180 10

7. (a) Implement the following using single IC 74151 for each function and some gates (if required). 10
   $F1 = \Sigma m(0,1,3,5,7)$
   $F2 = \Sigma m(1,4,5,7,8,12,13,15)$

   (b) Show how a 74148, 8-line to 3-line encoder, can be used with a 74138, 3:8 decoder to save wires in transmitting information from point A to point B in a digital system. 10

30 Dec 2010

P4-Con No-11

S.E. Electronics Sem III

Engineering Maths III

Con. 5531–10. **GT–6216**

(3 Hours) **[Total Marks : 100**

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1. (a) Find $L\left[t\sqrt{1+\sin t}\right]$. 05

(b) Find the Z-transform of Discrete Unit Step function,

$$U(k) = 1, \quad k \geq 0$$
$$= 0, \quad k < 0$$

Draw the graph of the given function and write the ROC. 05

(c) Determine $a.b, c$ & d so that the function $f(z) = (x^2 + axy + by^2) + i(cx^2 + dxy + y^2)$ is analytic. 05

(d) If $f(x) = C_1\phi_1(x) + C_2\phi_2(x) + C_3\phi_3(x)$, where C_1, C_2, C_3 are constants and ϕ_1, ϕ_2, ϕ_3 are orthonormal functions on (a, b), show that $\int_a^b [f(x)]^2 \, dx = C_1^2 + C_2^2 + C_3^2$. 05

2. **(a)** Derive Cauchy-Riemann equations in polar coordinates and find p if $f(z) = r^2 \cos 2\theta + ir^2 \sin p\theta$ is analytic. 06

(b) Show that $\{\cos x, \cos 2x, \cos 3x, \ldots\}$ is a set of orthogonal functions over an interval $(-\pi, \pi)$. Hence construct the corresponding set of orthonormal functions. 06

(c) Verify Greens's theorem for $\overline{F} = x^2 i - xy j$ and C is the triangle with vertices $A(0, 2)$, $B(2, 0)$, $C(4, 2)$. 08

3. **(a)** Find the Laplace transform of each of the following:- 06

(i) $\int_0^t u \cos^2 u \, du$ (ii) $te^{3t} erf\sqrt{t}$

(b) Find complex form of Fourier series for $f(x) = e^{-x}$, $-\pi < x < \pi$. 06

(c) Find non-singular matrices P and Q such that PAQ is in normal form. 08
Hence find (i) rank A (ii) A^{-1},

where $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

4. (a) Solve the system of equations $x + 2y - z = 1$, $x + y + 2z = 9$, $2x + y - z = 2$. 06

(b) Find the inverse Laplace transform of the following:- 06

(i) $\frac{e^{4-3s}}{(s+4)^{5/2}}$ (ii) $\frac{8e^{-3s}}{s^2+4}$.

(c) If $f(x)$ is periodic with period 2π and $f(x) = -\pi, \quad -\pi < x < 0$
$= x, \quad 0 < x < \pi,$ 08

obtain Fourier Series of $f(x)$ in $(-\pi, \pi)$. Deduce – $\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} = \frac{\pi^2}{8}$.

5. (a) Using Convolution theorem, find inverse Laplace transform of $\frac{1}{s^2(s+1)^2}$. 06

(b) Find the analytic function and its imaginary part if real part is $\frac{\sin 2x}{\cosh 2y + \cos 2x}$. 06

(c) Prove that $\overline{F} = (y^2 \cos x + z^3)\, i + (2y \sin x - 4)\, j + (3xz^2 + 2)\, k$ is a conservative field. Find (i) scalar potential for $\overline{F}$ (ii) the work done in moving an object in this field from $(0, 1, -1)$ to $((\pi/2, -1, 2)$. 08

6. (a) Using Laplace transformation, solve the following equation 06

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} + 8y = 1 \quad , \qquad y = 0 \text{ and } \frac{dy}{dt} = 0 \text{ at } t = 0.$$

(b) Find orthogonal trajectories of u=constant where $u = x^2 - y^2 - 2xy + 2x - 3y$. 06

(c) Find the inverse Z-transform of – 08

$$F(z) = \frac{1}{(z-3)(z-2)}$$

if ROC is (i) $|z| < 2$ (ii) $2 < |z| < 3$ (iii) $|z| > 3$.

7. (a) Evaluate $\int_0^{\infty} t\, e^{-3t} \sin t\, dt$ by using Laplace transform. 06

(b). Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = i, 0, -i$. Also determine the image of $|z| < 1$ under this transformation. 06

(c) Express the function – 08

$$f(x) = 1, \quad |x| \le 1$$
$$= 0, \quad |x| > 1$$

as a Fourier integeral and hence evaluate – $\int_0^{\infty} \frac{\sin w \cos wx}{w}\, dw$.

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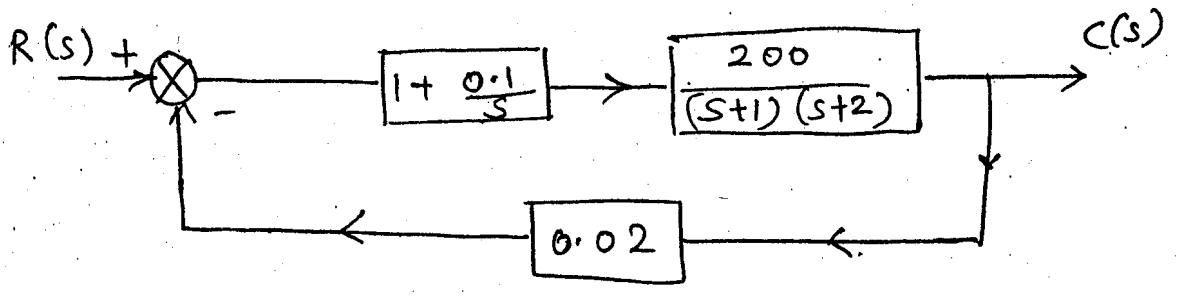
4. (a) Obtain the state variable model of the system whose transfer function is **10**

$$T(s) = \frac{s^2 + 3s + 3}{s^3 + 2s^2 + 3s + 1}.$$

(b) The control system is shown below. If the input to the system is (i) unit step (ii) unit ram. Determine e_{ss}. Also find the error constants. **10**



5. (a) Sketch the Bode plot and determine GM and PM for the unity feedback system having open loop transfer function $G(s) = \frac{10}{s(1+0\cdot5s)(1+0\cdot1s)}$. **15**

(b) Explain the effect of feedback on stability. **5**

6. (a) Sketch the Polar plot for the following (any **two**) :– **10**

(i) $G(s)\,H(s) = \frac{1}{s(1+Ts)}$

(ii) $G(s)\,H(s) = \frac{1}{(1+T_1s)(1+T_2s)}$

(iii) $G(s)\,H(s) = \frac{1}{s^2(1+Ts)}$

(b) Draw the Nyquist plot for a given open loop transfer function and test the stability **10**

$$G(s)\,H(s) = \frac{4s+1}{s^2(s+1)(2s+1)}$$

7. Write short notes on the following :– **20**
   (a) Co-relation between time domain and frequency domain specifications.
   (b) PI and PD controllers.
   (c) Mason's gain formula and its application.
   (d) Stepper Motors.

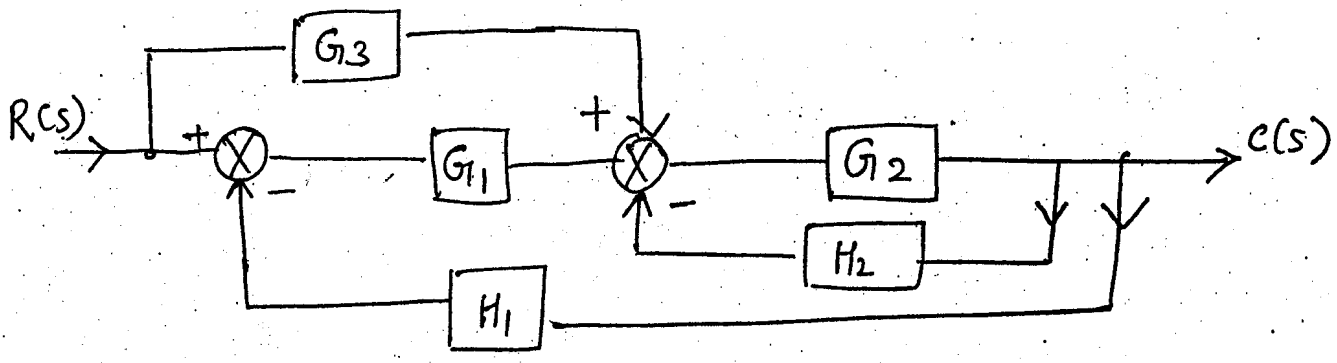
(3 Hours) [Total Marks : 100

S.E. - ETRX - Sem - III / Control System

N.B. : (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** question out of remaining **six** questions.
(3) Assume **suitable** data wherever **necessary**.
(4) **Figures** to the **right** indicate **full** marks.

1. Attempt following questions :– **20**
   (a) Define and explain the following terms related to the transfer function of a system.
      (i) Poles (ii) Zeros (iii) Characteristic equation
      (iv) Pole zero plot (v) order.
   (b) Derive the transfer function of simple closed loop system.
   (c) Explain the effect of addition of open loop poles and zeros to the function G(s) H(s).
   (d) Derive the expression for maximum peak overshoot of a standard second order control system.

2. (a) Find the transfer function $\frac{C(s)}{R(s)}$ using Block diagram reduction Technique. **10**



   (b) A unity feedback system is characterised by an open-loop transfer function **10**
   $G(s) = \frac{K}{s(s+10)}$. Determine the gain K so that system will have a damping ratio of 0·5, for this value of K determine settling time, peak overshoot and time to peak overshoot for a unit step input.

3. (a) Sketch the root locus for a given unity feedback control system whose open **10**
   loop transfer function is $G(s) = \frac{K}{s(s+4)\left(s^2+4s+20\right)}$
   Comment on the stability of system.
   (b) The open-loop transfer function of a unity feedback control system is given by **10**
   $G(s) = \frac{K}{(s+2)(s+4)\left(s^2+6s+25\right)}$. By applying the Routh Criterion, discuss the stability of the closed system as a function of K. Determine. the value of K which will cause sustained oscillations in the closed loop system. What is the corresponding oscillating frequency ?

S.E. Electronic / Sem III

P4-Con No-11

Con. 5531-10. Engineering Math III **GT-6216**

(3 Hours) **[Total Marks : 100**

N.B. : (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions out of remaining **six** questions.
(3) **Figures** to **right** indicates the **full** marks.

1. (a) Find $L\left[t\sqrt{1+\sin t}\right]$. 05

(b) Find the Z-transform of Discrete Unit Step function,

$$U(k) = 1, \quad k \geq 0$$
$$= 0, \quad k < 0$$

Draw the graph of the given function and write the ROC. 05

(c) Determine $a.b, c$ & d so that the function $f(z) = (x^2 + axy + by^2) + i(cx^2 + dxy + y^2)$ is analytic. 05

(d) If $f(x) = C_1\phi_1(x) + C_2\phi_2(x) + C_3\phi_3(x)$, where C_1, C_2, C_3 are constants and ϕ_1, ϕ_2, ϕ_3 are orthonormal functions on (a, b), show that $\int_a^b [f(x)]^2 \, dx = C_1^2 + C_2^2 + C_3^2$. 05

2. (a) Derive Cauchy-Riemann equations in polar coordinates and find p if $f(z) = r^2 \cos 2\theta + ir^2 \sin p\theta$ is analytic. 06

(b) Show that $\{\cos x, \cos 2x, \cos 3x, \ldots\}$ is a set of orthogonal functions over an interval $(-\pi, \pi)$. Hence construct the corresponding set of orthonormal functions. 06

(c) Verify Greens's theorem for $\bar{F} = x^2 i - xy j$ and C is the triangle with vertices $A(0, 2)$, $B(2, 0)$, $C(4, 2)$. 08

3. (a) Find the Laplace transform of each of the following:- 06

(i) $\int_0^t u \cos^2 u \, du$ (ii) $te^{3t} erf\sqrt{t}$

(b) Find complex form of Fourier series for $f(x) = e^{-x}$, $-\pi < x < \pi$. 06

(c) Find non-singular matrices P and Q such that PAQ is in normal form. 08
Hence find (i) rank A (ii) A^{-1},

where $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

4. (a) Solve the system of equations $x + 2y - z = 1$, $x + y + 2z = 9$, $2x + y - z = 2$. 06

(b) Find the inverse Laplace transform of the following:- 06

(i) $\frac{e^{4-3s}}{(s+4)^{5/2}}$ (ii) $\frac{8e^{-3s}}{s^2+4}$.

(c) If $f(x)$ is periodic with period 2π and $f(x) = -\pi, \quad -\pi < x < 0$ $= x, \quad 0 < x < \pi,$ 08

obtain Fourier Series of $f(x)$ in $(-\pi, \pi)$. Deduce – $\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} = \frac{\pi^2}{8}$.

5. (a) Using Convolution theorem, find inverse Laplace transform of $\frac{1}{s^2(s+1)^2}$. 06

(b) Find the analytic function and its imaginary part if real part is $\frac{\sin 2x}{\cosh 2y + \cos 2x}$. 06

(c) Prove that $\overline{F} = (y^2 \cos x + z^3)\, i + (2y \sin x - 4)\, j + (3xz^2 + 2)\, k$ is a conservative field. Find (i) scalar potential for $\overline{F}$ (ii) the work done in moving an object in this field from (0, 1, −1) to ((π/2, −1, 2). 08

6. (a) Using Laplace transformation, solve the following equation 06

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} + 8y = 1 \quad , \quad y = 0 \text{ and } \frac{dy}{dt} = 0 \text{ at } t = 0.$$

(b) Find orthogonal trajectories of u=constant where $u = x^2 - y^2 - 2xy + 2x - 3y$. 06

(c) Find the inverse Z-transform of – 08

$$F(z) = \frac{1}{(z-3)(z-2)}$$

if ROC is (i) $|z| < 2$ (ii) $2 < |z| < 3$ (iii) $|z| > 3$.

7. (a) Evaluate $\int_0^{\infty} t\, e^{-3t} \sin t\, dt$ by using Laplace transform. 06

(b). Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = i, 0, -i$. Also determine the image of $|z| < 1$ under this transformation. 06

(c) Express the function – 08

$$f(x) = 1, \quad |x| \le 1$$
$$= 0, \quad |x| > 1$$

as a Fourier integeral and hence evaluate – $\int_0^{\infty} \frac{\sin w \cos wx}{w}\, dw$.