S.E. IT. sem III old Nov-13 25/11/13

sub - A.M.-III

28-10-2013-DTP-P-7-MU-7

Con. 5398-13. **LJ - 10264**

(3 Hours) **[Total Marks : 100**

N. B. : (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions from the remaining **six** Questions.
(3) Assume **suitable data,** if necessary and justify the same.

1. (a) Show that $f(z) = z^n$ is an analytic function, hence find $f^1(z)$ **5**

(b) Show that every square matrix A can be uniquely expresented as the sum of Hermition and skew-Hermition matrix. **5**

(c) Evaluate $\oint_C \frac{e^{3z}}{z^3}.dz$ where C is $|z| = 1$. **5**

(d) State and prove first shifting property, hence find L $\{e^{-at}$ cost bt$\}$ **5**

2. (a) Prove that the function $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is harmonic and find the corresponding analytic function. **6**

(b) Find Laplace transform of **6**

(i) t Sin3tcos5t (ii) $\frac{1-\cos at}{t}$

(c) Find fourier series for $f(x)=x^2$ in $(0,2\Pi)$ **8**

hence deduce that $\frac{\Pi^2}{12} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots$

3. (a) Reduce the following matrix to normal form, and find its rank. **6**

$$\begin{bmatrix} 4 & 3 & 0 & -2 \\ 3 & 4 & -1 & -3 \\ 7 & 7 & -1 & -5 \end{bmatrix}$$

(b) If $f(z) = u + iv$ is an analytic function of z, and $u - v = e^x$ (cosy-siny), find f(z) in terms of z. **6**

(c) Find inverse Laplace transform of **8**

(i) $\frac{s}{(s+1)(s^2+1)}$ (ii) $\tan^{-1}(2/s)$

[TURN OVER

4. (a) Show that the set s = {sinx, sin3x, sin5x ....} is orthogonal over [0, ∏/2], find the corresponding orthonormal set. **6**

(b) Find fourier series for **6**

$$f(x) = x + \Pi \quad -\Pi \le x \le -\Pi/2$$
$$= \Pi/2 \quad -\Pi/2 \le x \le \Pi/2$$
$$= \Pi - x \quad \Pi/2 \le x \le \Pi$$

(c) For what value of λ the equations x + y + z = 1; x + 2y + 4z = λ, x + 4y + 10z $=\lambda^2$ have a solution and solve them completely in each case. **8**

5. (a) Prove that the matrix $A = 1/9\begin{bmatrix} -2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & -2 & 2 \end{bmatrix}$ is orthogonal and hence find A^{-1}. **6**

(b) By casing convolution theom, find inverse Laplace transform of $\frac{s}{(s^2+a^2)(s^2+b^2)}$ **6**

(c) If f(z) is an analytic function of z, prove that **8**

$$\left(\frac{\partial^2}{ox^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^P = P^2|f(z)|^{P-2}|f^1(z)|^2$$

6. (a) Using cauchy's residue theorem, evaluate **6**

$$\oint_C \frac{1-e^{2Z}}{Z^4}dz \quad \text{where C is } |z|=1$$

(b) Find half range fourier sine series for f(x) = x sinx in (0,∏) **6**

(c) Evaluate $\left(\frac{d^2y}{dt^2}+2\frac{dy}{dt}+3y\right) = e^{4t};$ **8**

y (0) = 0, y^1 (0) = 1, by using Laplace transform

7. (a) Find non-singular matrices P and Q, such that PAQ is in normal form, where **6**

$$A=\begin{bmatrix} 1 & 5 & 6 & 11 \\ 3 & 7 & 10 & 17 \\ 4 & 8 & 12 & 20 \end{bmatrix}$$

(b) Evaluate $\int_0^{2\Pi} \frac{d\theta}{13+5\cos\theta}$ by using cauchy's residue theorem. **6**

(c) Expand all possible Taylor's series and Laurentz series for $f(z)=\frac{z}{(z+2)(z+3)}$ about $z = 1$. **8**
