Con. 2611-05. (REVISED COURSE) AM-9679

(3 Hours) [Total Marks : 100

N.B. Answer any **five** questions.

1. (a) (i) With the help of a Venn diagram show that the conditonal probability of occurance of an event A given that the event B has accured, is given by — 5

$$P(A|B) = \frac{P(AB)}{P(B)}.$$

(ii) An Urn contains two black balls and three white balls. Two balls are selected at random from the urn without replacement and sequence of colours is noted. Find the probability that both balls are black. 5

(b) Suppose that 5 cards to be drawn at random from a standard deck of 52 cards. If all the cards drawn are red, what is the probability that all of them are hearts. 10

2. (a) Explain the concept of a continuous random variable. Bring even that it is actually a function in the conventional sense with a domain and a range. Explain how a probability is assigned to such a random variable. Take the help of a mapping diagram. 12
Explain what is a cumulative distribution function and what is a probability density function.

(b) The transmission time X of messages in a communication system obey the exponential probability law with a parameter λ, that is — 8
$p[X > x] = e^{-\lambda x}, \; x > 0.$
Find cdf of X that is $F(x) = p(X \le x)$ and pdf of X and sketch them as functions of x.

3. (a) Explain how the characteristic function of a random variable $\phi_X(w)$ of X is defined ? Show that — 14

$$\left.\frac{d}{dw}\phi_X(w)\right|_{w=0} = jE[X] \text{ in geneal, } \left.\frac{d^n}{dw^n}\phi_X(w)\right|_{w=0} = j^n E[X^n].$$

(b) An exponential distributed random variable X, with parameter λ is given by $X = \lambda e^{\lambda x}$. Find $E[X] = \phi'_X(0)/j$. 6

4. (a) We define conditional cdf of Y given X = x by 8
$F_y(y|x) = \lim_{h \to 0} F_y(y | x < X \le x + h)$ and applying Baye's rule it can be written as

$$\lim_{h \to 0} F_y(y | x < X \le x+h) = \frac{P[Y \le y, \; x < X \le x+h]}{P[x < X \le x+h]}.$$

Show that $f_y(y|x) = \frac{d}{dy} F_y(y|x) = \frac{f_{xy}(x, y)}{f_x(x)}$.

(b) X and Y are two continuous random variables. Then joint probability density function is given by — 12

$$f_{x,y}(x,y) = \begin{cases} c\,e^{-x}e^{-y}, & 0 \le y \le x < \infty \\ 0, & \text{elsewhere} \end{cases}$$

(i) Find the value of normalization constant C and
(ii) $f_x(x)$, (iii) $F_y(y)$, (iv) $F_x(x|y)$, (v) $f_y(y|x)$, (vi) $E(Y|x)$ (vii) $E(X|y)$.
Where last two are conditional expectation of X and Y.

5. (a) Let X and Y two continuous random variables. 10
(i) Derive an expression for their joint moment at the origin. Why it is called correlation ? Explain its physical significance.
(ii) Derive an expression for their joint central moment. Why it is called covariance ? Explain its physical significance.
(iii) Derive an expression for their normalized covariance. Why it is called covariance coefficient ? Explain what is its physical significance ? What is its range of values ?
(iv) Explain when X and Y are orthogonal, when they are independent and they are uncorrelated.

(b) Random variables X and Y have joint density function — 10

$$f_{XY}(x, y) = \begin{cases} (x+y)^2/40 & -1 < x < 1, \; -3 < y < 3 \\ 0 & \text{elsewhere} \end{cases}$$

(i) Find marginal densities of X and Y.
(ii) Find mean and variance of X and Y.
(iii) Find second order moment of X and Y.
(iv) Find correlation coefficient of X and Y.

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6. (a) Explain what is a rando ı process. Show how a random process can be described by an indexed set of random variables. Hence define ensemble mean, auto-correlation and auto-covariance of the process in terms of the indexed random variables in usual mathematical forms. **12**
What is a stationary random process ? A random process is given by $X(t) = A \cos 2\pi t$, where A is some random variable. Is the process stationary ?
(b) What is an Ergodic random process ? Explain. Explain in details why time averages and time autocorrelation functions of sample functions of an ergodic process are random variables over the ensemble. Show that the expectations of these random variables (i.e. time averages and time autocorreation tunction) are equal to time averages and time autocorrelation respectively of the sample function itself. **8**

7. (a) What is meant by a wide-sentionary random process ? What are the conditions it must satisfy to be an wide-sense stationary process ? Explain. Show that the process X(t) is wide sense stationary. X(t) is given by $X(t) = A \cos (w_0 t + \theta)$. **10**
Where A and w_0 are constants and θ is an uniformly distributed random variable in the interval [0, 2π] with a probability density function.

$$f_\theta(0) = \begin{cases} \frac{1}{2\pi}, & 0 < \theta \le 2\pi \\ 0, & \text{elsewhere} \end{cases}$$

(b) Find power spectrum of the process $X(t) = A \cos (w_0 t + \theta)$ as specified above. **10**