S.E (COMPUTER INFORMATION TECHNOLOGY) (SEM III) (REV)
EXAMINATION, OCTOBER, 2006
Applied mathematics III

19/6/06

Con. 4812-06.

YM-5248

(REVISED COURSE)

(3 Hours)

[Total Marks : 100

N.B. (1) Question No. 1 is **compulsory**.
(2) Attempt any **four** questions out of remaining **six** questions.
(3) **Figures** to the **right** indicate **full** marks.
(4) Answers to the sub-questions of main question must be written **together**.

1\. (a) If $A = \begin{bmatrix} 2 & 1 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ verify that A (adj. A) = | A | I. 5

(b) Find $L\left\{ \text{erf}\sqrt{t} \right\}$. 5

(c) Find the image of the strip $2 \leq x \leq 4$ in the z-plane under the transformation $w = \dfrac{2}{z}$. 5

(d) Obtain the half range sine series for— 5

$$f(x) = x, \; 0 < x < 1$$
$$= 2 - x, \; 1 < x < 2$$

hence deduce that—

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots$$

2\. (a) Find the Fourier series for, 6

$f(x) = \frac{1}{2}(\pi - x)$ in $(0, 2\pi)$. Hence deduce that,

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots$$

(b) **Evaluate the integral by using L.T.** 6

$$\int_0^\infty t^{-1} e^{-t} \left[\int_0^t e^{-u} \sin u \, du \right] dt.$$

(c) Use L.T. to solve $(D^2 + D - 2)x = 2(1 + t - t^2)$, $x = 0$, $Dx = 03$ for $t = 0$. 8

3\. (a) Test for consistency the following equations and solve them if consistent. 6

$$x_1 - x_2 + x_3 - x_4 + x_5 = 1, \quad 2x_1 - x_2 + 3x_3 + 4x_5 = 2.$$
$$3x_1 - 2x_2 + 2x_3 + x_4 + x_5 = 1, \quad x_1 + x_2 + 2x_4 + x_5 = 0.$$

(b) Show that the function, 6

$$f(z) = \frac{x^2 y^2 (x + iy)}{x^4 + y^{10}}, \quad z \neq 0.$$
$$= 0, \qquad z = 0$$

is not analytic but C-R equations are satisfied at origin.

(c) Find the Fourier series for, 8

$f(x) = |\cos x|, \; -\pi < x < \pi$.

4\. (a) Examine whether the vectors 6

$x_1 = [1, 1, 1, 3]$, $x_2 = [1, 2, 3, 4]$
$x_3 = [2, 3, 4, 7]$ are dependent or not.

It those are dependent find relation between them.

(b) Find the L.T. 6

(i) $t^3 \sinh^2 t$, (ii) If $L\left\{ 2\sqrt{\frac{t}{\pi}} \right\} = \frac{1}{s^{3/2}}$, s.t. $L\left\{ \frac{1}{\sqrt{\frac{1}{\pi t}}} \right\} = \frac{1}{\sqrt{s}}$.

(c) If $W = \varphi + i\Psi$ represents complex potential for an electric field then show that, 8

$$\Psi = x^2 - y^2 + \frac{x}{x^2 + y^2}$$ is harmonic. Determine φ and W.

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5. (a) Find the inverse L.T. 6

(i) $\cot^{-1}(s+1)$, (ii) $\dfrac{s}{\left(s^2+16\right)^2}$ (use convolution Th^m).

(b) If f(z) is analytic function. Prove that, 6

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]|f(z)|^2 = 4\left|f^1(z)\right|^2.$$

(c) Express the function— 8

$$\begin{aligned} f(x) &= 0, \quad x<0 \\ &= \sin x, \; 0<x<\pi \\ &= 0, \quad x>\pi \end{aligned}$$

as Fourier integral and hence deduce that—

$$\int_0^\infty \frac{\cos\frac{\lambda\pi}{2}}{1-\lambda^2}\,d\lambda = \frac{\pi}{2}.$$

6. (a) Find non-singular matrices P and Q such that PAQ is normal form. Hence find rank 6

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$$

(b) Show that the functions $f_1(x)=1$, $f_2(x)=x$ are othogonal on $(-1, 1)$. Determine the constants a and b such that the function $f_3(x) = -1+ax+bx^2$ is orthogonal to both $f_1(x)$ and $f_2(x)$ on the same interval. 6

(c) If $f(z) = u + i\upsilon$ is an analytic function then s.t. the curves $u = c_1$ and $v = c_2$ are orthogonal. 8
If $u = x^3y - xy^3 = c$ find orthogonal trajectory.

7. (a) Express the function in heavisides unit step frequency and hence find L.T. 6

$$\begin{aligned} f(t) &= \cos t, \; 0<t<\pi \\ &= \cos 2t, \; \pi<t<2\pi \\ &= \cos 3t, \; t>2\pi. \end{aligned}$$

(b) Find the bilinear transformation which maps $z=\infty, i, 0$ onto the points, $w = 0, i, \infty$. Hence find the fixed points. 6

(c) Show that every square matrix A can be uniquely expressed as p + i Q whee P & Q are Hermitian matrices. Hence express— 8

$$\begin{bmatrix} 2i & -3 & 1-i \\ 0 & 2+3i & 1+i \\ -3i & 3+2i & 2-5i \end{bmatrix} \text{ in } P+iQ \text{ form.}$$