S.E. Etrx Hdelcomm^n Applied Maths-III 3-6-06

41 : 1sthalf-D98

Con. 1934-06. **(REVISED COURSE)** **TV-7929**

(3 Hours) **[Total Marks : 100**

N.B. (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** out of remaining **six** questions.
(3) Assume any **suitable** data, wherever **required** but justify the **same**.
(4) **Figures** to the **right** indicate **full** marks.

1. (a) State and prove change of scale property. If $L\left(\operatorname{erf}\sqrt{t}\right)=\frac{1}{s\sqrt{s+1}}$ find $L\left\{\operatorname{erf} 2\sqrt{t}\right\}$. 5

(b) Express $A=\begin{bmatrix} 2i & 2+i & 1-i \\ -2+i & -i & 3i \\ -1-i & 3i & 0 \end{bmatrix}$ as P + iQ. Where P real and skew symmetric. Q real and symmetric. 5

(c) Find the map of line $y-x+1=0$ by transformation $w=\frac{1}{z}$. 5

(d) Find Fourier series for $f(t)=1-t^2$ when $-1 \le t \le 1$. 5

2. (a) If $f(t) = t+1$ for $0<t<2$
$\qquad\quad\; 3$ for $t>2$
find $L\{f(t)\}$, $L\{f'(t)\}$ and $L\{f''(t)\}$. 7

(b) Find $L^{-1}\left\{\frac{s+2}{\left(s^2+4s+8\right)^2}\right\}$. 5

(c) Find the Fourier expansion for $f(x)=\sqrt{1-\cos x}$ in $(0, 2\pi)$. Hence deduce that $\frac{1}{2}=\sum_{n=1}^{\infty}\frac{1}{4n^2-1}$. 8

3. (a) Using Convolution Theorem find $L^{-1}\left\{\frac{1}{(s-2)(s+2)^2}\right\}$ and verify the result by Laplace transform. 6

(b) Solve the following equation by using Laplace transform $\frac{dy}{dt}+2y+\int_0^t y\,dt=\sin t$ given that y(0) = 1. 6

(c) If A is non-singular matrix of order n prove that A adj A = | A | I and for matrix A verify that 8

A adj A = | A | I. Where $A=\begin{bmatrix} 2 & 1 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$.

4. (a) If f(t) is a periodic function of period a, show that — 5

$$L\{f(t)\}=\frac{1}{1-e^{-as}}\int_0^a e^{-st} f(t)\,dt.$$

(b) Find Fourier series for $f(x)=x^2$ for $(-l, l)$ hence deduce that — 7

$$\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots=\frac{\pi^2}{12}$$

$$\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots=\frac{\pi^2}{6}.$$

(c) (i) If $A=\frac{1}{3}\begin{bmatrix} 1 & 2 & a \\ 2 & 1 & b \\ 2 & -2 & c \end{bmatrix}$ is orthogonal find a, b, c and find A^{-1}. 6

(ii) Prove that A is an orthogonal then $|A| = \pm 1$. 2

5. (a) Prove that $L\{f(t)H(t-a)\} = e^{-as} L\{f(t+a)\}$ and hence evaluate $L\left[\left\{1+2t-t^2+t^3\right\} H(t-2)\right]$. 6

(b) If the following system has nm trivial solution then p.t. $a + b + c = 0$ or $= a = b = c$ 5

$$ax + by + cz = 0$$
$$bx + cy + az = 0$$
$$cx + ay + bz = 0.$$

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(c) (i) If $u = \dfrac{\sin 2x}{\cosh 2y + \cos 2x}$ find corresponding analytic function. **5**

(ii) If $f(z) = u + iv$ S.T. $|f'(z)|^2 = \begin{vmatrix} \dfrac{\partial u}{\partial x} & \dfrac{\partial u}{\partial y} \\ \dfrac{\partial v}{\partial x} & \dfrac{\partial v}{\partial y} \end{vmatrix}$. **4**

6. (a) Find orthogonal family of curves for function $e^{-x} \cos y + xy =$ constant in (x, y) plane. **6**

(b) If $f(x) = x$ in the interval $(0, 2)$ find half range cosine series using Parseval's identity and deduce that — **8**

$$\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots\ldots$$

(c) Prove that set of functions $\{1, \sin x, \cos x, \sin 2x, \cos 2x, \ldots\}$ is orthogonal over $(0, 2\pi)$ and construct a corresponding orthonormal set. **6**

7. (a) Find bilinear transformation which maps the pts. $Z = 1, i, -1$ into pts $W = i, 0, -i$ respectively. Hence find image of $|z| < 1$ and find invariant pts of this transformation. **10**

(b) Find the rank of matrix by reducing it to normal form — **5**

$$\begin{bmatrix} 1 & -1 & -2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 1 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}.$$

(c) Prove that matrix $\begin{bmatrix} \dfrac{1+i}{2} & \dfrac{-1+i}{2} \\ \dfrac{1+i}{2} & \dfrac{1-i}{2} \end{bmatrix}$ is unitary and find A^{-1}. **5**