

- N.B.** (1) Question No. 1 is compulsory.
 (2) Attempt any four out of remaining six questions.
 (3) Figures to the right indicate marks.

1. (a) Evaluate $I = \int_C (z - z^2) dz$ where C is upper half of circle $|z| = 1$. What will be the value of I for lower half of the same circle? State the Theorem used. 5

- (b) If $A = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$ prove that $3 \tan A = A \tan 3$. 5

- (c) State Cauchy's Residue Theorem and use it to find value of — 5

$$I = \oint_C \frac{z-1}{(z+1)^2(z-2)} dz$$

Where C encloses both the poles of $f(z)$.

- (d) Solve by Gauss-Jordan Reduction. 5

$$x + y + z = 9$$

$$2x - 3y + 4z = 13$$

$$3x + 4y + 5z = 40$$

2. (a) (i) If λ is Eigen value of a square matrix A then prove that λ^n is an eigen value of A^n . 2

- (ii) Find Eigen values and vectors for A^3 if $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$. 6

Is A derogatory?

- (b) Find the real root of the equation $x^3 - 2x - 5 = 0$ using Newton-Raphson Method after four approximations. 6

- (c) With Usual Notations prove. 6

$$(1) \mu^2 = 1 + \frac{1}{4} \delta^2$$

$$(2) \mu \delta = \text{Sin h}(hD).$$

3. (a) Solve using Gauss-Seidal Iteration Method. 6

$$6x + y + z = 105$$

$$4x + 8y + 3z = 155$$

$$5x + 4y - 10z = 65.$$

- (b) Use Lagrange's formula to find velocity of the particle as $V = f(t)$ at $t = 3.5$ from table. 6

t	0	1	2	3
v	21	15	12	10

- (c) Use Residue Theorem to Evaluate :— 8

$$(1) \int_0^{2\pi} \frac{d\theta}{3 + 2 \text{Cos } \theta}$$

$$(2) \int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2 + a^2)(x^2 + b^2)}$$

4. (a) Use Taylor's series method to solve the differential equation —

6

$$\frac{dy}{dx} = -xy, \quad y(0) = 1.$$

(b) Obtain the function $f(x)$ whose first difference is $x^3 + 3x^2 + 5x + 12$.

6

(c) (i) Show that the Eigen values of A are of unit modulus.

8

$$A = \begin{bmatrix} \frac{1+i}{2} & \frac{-1+i}{2} \\ \frac{1+i}{2} & \frac{1-i}{2} \end{bmatrix}.$$

(ii) Use C-H Theorem to find A^{-1} for

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}.$$

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5. (a) If $f(p) = \oint_C \frac{4z^2 + z + 4}{z - p} dz$. Where C is ellipse $4x^2 + 9y^2 = 36$. Find the values of $f(4)$, $f(1)$, $f'(-1)$, $f''(5)$. 6

(b) Solve $\frac{dy}{dx} = x + 3y$ with $x_0 = 0$, $y_0 = 1$ by Euler's modified rule and determine y at $x = 0.05$ in one step. 6

(c) Show that the matrix A is diagonalisable. Find diagonal matrix and transforming matrix. 8

$$A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$$

6. (a) (i) Expand $f(z) = z^3 e^{1/z}$ about $z = 0$ and find the residue at $z = 0$. 6

(ii) Show that $f(z) = \frac{\sin z}{z}$ has a removable singularity at $z = 0$.

(b) If $y = f(x)$ is a polynomial of 7th degree and 6

$$y_0 + y_8 = 1.9243$$

$$y_1 + y_7 = 1.9590$$

$$y_2 + y_6 = 1.9823$$

$$y_3 + y_5 = 1.9956$$

Find y_4 .

(c) Use Runge-Kutta Method of order 4 to find $y(0.2)$ in two steps for $\frac{dy}{dx} = \sqrt{x+y}$, $y(0) = 1$. 8

7. (a) Obtain Laurent's series expansion for $f(z) = \frac{2z-3}{z^2-4z-3}$ in powers of $(z-4)$ in the region $|z-4| < 1$. 6

(b) Prove that Eigen values of Hermitian matrix are real. 6

(c) Define interpolation. 8

Use Newton's forward difference interpolation formula to find number of persons getting wages less than Rs. 15.

Wages in Rs.	0 - 10	10 - 20	20 - 30	30 - 40
No. of Persons	09	30	35	42