(3 Hours) K-TExam **[Total Marks : 100**

N.B. : (1) Question No. 1 is compulsory.
(2) Answer any four out of remaining six questions.
(3) Assumptions made should be clearly stated.
(4) Assume any suitable data wherever required but justify the same.
(5) Figures to the right indicate full marks.
(6) Illustrate answerers with sketches wherever required.
(7) Answers to the questions should be grouped and written together i.e., all answers to sub-questions of individual questions should be answered one below the other.
(8) Use legible handwriting. Use a Blue/Black ink pen to write answers. Use of pencil should be done only to draw diagrams and graphs.

1(a) Find the distribution function for the sum of the numbers appearing on the toss of two unbiased dice. (4)

(b) Obtain the equation of the line of regression of cost on age from the following table giving the age of a car of certain make and the annual maintenance cost : (4)

Age of car	2	4	6	8
Maintenance	1	2	2.5	3

(c) In the usual notation prove that $\sigma^2_{X-Y} = \sigma^2_X + \sigma^2_Y - 2r_{XY}\sigma_X\sigma_Y$ (4)

(d) If 10 points are chosen from the interior of an equilateral triangle of side 1 unit show that there must be at least two points less than 1/3 rd unit apart. (4)

(e) Given the posets $(D_4, \leq)$ and $(D_9, \leq)$ under the usual notation draw the Hasse diagram for $L = D_4 \times D_9$ under the product partial order. (4)

2(a) A population consists of five numbers 2,3,6,8 and 11. Consider all possible samples of size two that can be drawn with replacement from this population. (12)
Find the mean and variance of
(i) the population
(ii) the sampling distribution of means
Verify (ii) directly from (i) by use of suitable formulas.

(b) It is known that the probability of an item produced by a certain machine will be defective is 0.05. If the produced items are sent to the market in packets of 20, find the number of packets con ining at least, exactly and at most 2 defective items in a consignment of 1000 packets (8)
(i) using the binomial distribution
(ii) using the Poisson approximation to the binomial distribution.

3(a) If R(t) = log M(t) where M(t) is the m.g.f. about the origin of a discrete random variable then prove that $\mu = R'(0)$ and $\sigma^2 = R''(0)$ .Hence, find the mean and variance of the Poisson distribution. And then find $P(\mu - 2\sigma < X < \mu + 2\sigma)$ if $\mu = 4$. (10)

(b) Let X= {a,b,c}and Y={0,1}
(i) Write down all the equivalence relations on X. (2)
(ii) Draw all the distinct Hasse diagrams if X is a poset. (2)
(iii) Find the number of distinct binary operations possible on Y to make it a semi-group. (3)
(iv) List all the functions from X to Y. (3)

4. A geneticist working for a seed company develops a new carrot for growing in heavy clay soil. After measuring 5000 of these carrots it can be said that carrot length X is normally distributed with mean μ =11.5 cm and σ = 1.15 cm.

(i) What is the probability that X will take on a value in the interval $10.0 \leq X \leq 13.0$? (4)
(ii) The seed company wants to state in its catalog that these new carrots "grow to between 10 cm and 13 cm". To do this, however, the company requires that at least 80% of the carrots are between 10 cm and 13 cm and that at least 90% are 10 cm or more. Can the company use this phrase ? (4)
(iii) What is the probability for a random sample of 25 of these carrots that the mean $\bar{X}$ of the sample will be within 0.5 cm of μ in either direction ? (4)
(iv) The company conducts a yearly quality control test to see, among other things, if it can still say that " the average length of the carrots from the seeds will be 11.5 cm" Assigned this test, you take a random sample of 40 of these carrots from a field of mature carrots and find that their average length is $\bar{x}$ = 11.8 cm. Do a two-tailed test at 0.05 level of significance. (4)
(v) There is reason to believe that the heavy soil carrots will not grow as long in sandy soil(i.e. will not average 11.5 cm). You test this by growing these carrots in sandy soil, taking a random sample of 50 mature carrots, and finding that their average length is $\bar{x}$=11.1 cm Do a left-tailed test at 0.01 level of significance. (4)

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5 (a) The following data represent the marks obtained by 12 students in 2 tests one held before coaching and the other after coaching. (10)

Test 1	55	60	65	75	49	25	18	30	35	54	61	72
Test 2	63	70	70	81	54	29	21	28	32	50	70	80

(i) Does the data indicate that the coaching was effective in improving the performance of the students.

(ii) Find the correlation coefficient between the two sets of marks.

(b) Prove that $(B,\wedge,\vee,-,0,1)$ is a Boolean algebra, where B = $\{0,1\}$ and $\vee$ and $\wedge$ are defined as $0 = 0\vee 0 = 0\wedge 0 = 0\wedge 1 = 1\wedge 0$ and $1 = 1\vee 1 = 1\wedge 1 = 0\vee 1 = 1\vee 0$. (4)

(c) Let S = {a,b,c,d } (6)

(i) Complete the binary operation $*$ in the operation table given below to make S a semi-group.

*	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	--	d	--	--
d	d	--	b	a

(ii) Show that R = {(a,a),(a,b),(b,a),(b,b),(c,c)(c,d),(d,c),(d,d)} defined on S is an equivalence relation

(iii) Find S/R and write the operation table for it and find the natural homomorphism g: S $\to$ S/R.

6. (a) Let $B_n = \{x = x_1x_2...x_n \mid x_i = 0 \, or \, x_i = 1 \, for \, 1 \le i \le n\}$

(i) Define ~ over B_n by x~y if and only if $x_i \le y_i$. Prove that $(B_n,\sim)$ is a lattice. (6)

(ii) Define $x * y = z = z_1z_2...z_n$ where $z_i = 1$ if $x_i \ne y_i$, and $z_i = 0$ if $x_i = y_i$. Prove that $(B_n,*)$ is an abelian group. (6)

(iii) Let $(B_n,*_1)$ and $(B_n,*_2)$ be two groups where $x *_1 y = z$ and $x *_2 y = u$, given by $z_i = 1$ and $u_i = 0$ if $x_i \ne y_i$, and $z_i = 0$ and $u_i = 1$ if $x_i = y_i$. Define $\psi : (B_n,*_1) \to (B_n,*_2)$ by $\psi(x) = v$ where $v_i = 0$ if $x_i = 1$, and $v_i = 1$ if $x_i = o$. Show that ψ is an isomorphism. (4)

(b) Show that the additive group Z_4 is isomorphic to the multiplicative group of non-zero elements of Z_5. (4)

7. (a) Theory predicts that the proportion of beans in 4 groups A,B,C and D should be 9:3:3:1. In an experiment among 1600 beans the numbers in the 4 groups were 882,313,287 and 118 respectively. Does the experiment support the theory ? (6)

(b) Under what conditions is (5)

(i) a sample said to be a random sample

(ii) a random experiment said to have a binomial distribution

(iii) a function f(x) said to be a probability density function

(iv) a binomial distribution approximated by a normal distribution

(v) a sample statistic said to be a good estimator

(c) Prove that $(Z_5,+_5,\times_5)$ is a ring. Is it an integral domain ? Is it a field ? (9)