

(3 Hours)

[Total Marks : 100

- N.B. : (1) Question No. 1 is compulsory.
 (2) Attempt any four questions out of the remaining six questions.
 (3) Figures to the right indicate full marks.
 (4) Answers to the sub-questions of main question must be written together.

1. (a) If $\int_0^{\infty} e^{-2t} \sin(t + \alpha) \cos(t - \alpha) dt = \frac{3}{8}$ then find α . 20

- (b) Obtain complex form of fourier series for $f(x) = e^{ax}$ in $(-l, l)$
 (c) Find the orthogonal trajectory of the family of curves given by $3x^2 y - y^3 = C$.
 (d) Show that every square matrix can be uniquely expressed as sum of a Hermition matrix and a skew Hermition matrix.

2. (a) If $A = \frac{1}{3} \begin{bmatrix} 1 & 2 & a \\ 2 & 1 & b \\ 2 & -2 & c \end{bmatrix}$ is orthogonal . Find a, b, c. 6

- (b) Find non singular matrices P and Q such that PAQ is in normal form also find its rank. 6

$$A = \begin{bmatrix} 1 & 2 & -1 & -2 \\ -2 & 5 & 3 & 0 \\ 1 & 0 & 1 & 10 \end{bmatrix}$$

(c) Find (i) $L^{-1} \left\{ \frac{5s^2 - 15s - 11}{(s+1)(s-2)^2} \right\}$ 8

(ii) $L^{-1} \left\{ \tan^{-1} \left(\frac{2}{5} \right) \right\}$

3. (a) Obtain Fourier Series for function— 6

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \leq x \leq 0 \\ 1 - \frac{2x}{\pi}, & 0 \leq x \leq \pi \end{cases}$$

hence deduce that— $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$

- (b) Find (i) $L \left\{ t \sqrt{1 + \sin 2t} \right\}$ 6

(ii) Find L.T. of $f(t)$
 where $f(t) = \sin 2t, 0 < t < \pi$
 $f(t) = 0 \quad t > \pi$

- (c) Use L.T. to solve $(D^2 + 2D + 5)y = e^{-t} \sin t$ when $y(0) = 0, y'(0) = 1$ 8

4. (a) Find Fourier series $f(x)$ —

6

$$f(x) = \frac{3x^2 - 6x\pi + 2\pi^2}{12} \text{ in } (0, 2\pi)$$

hence deduce that $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots$

- (b) Show that the functions $f_1(x) = 1$, $f_2(x) = x$ are orthogonal on $(-1, 1)$ determine the constants a and b such that $f_3(x) = -1 + ax + bx^2$ is orthogonal to both f_1 and f_2 on the same interval. 6

- (c) If $f(z) = u + iv$ is analytic and $u + v = \frac{2 \sin 2x}{e^{2x} + e^{-2y} - 2 \cos 2x}$ 8

Find $f(z)$.

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5. (a) Find the inverse of the matrix—

$$S = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \text{ and if } A = \frac{1}{2} \begin{bmatrix} 4 & -1 & 1 \\ -2 & 3 & -1 \\ 2 & 1 & 5 \end{bmatrix}$$

Show that SAS^{-1} is a diagonal matrix dig (2, 3, 1).

(b) Using Laplace transform to evaluate—

$$\int_0^{\infty} e^{-t} (1 + 2t - t^2 + t^3) H(t-1) dt.$$

(c) Find the bilinear transformation which maps the points 2, i, -2 on to the points 1, i, -1 hence find its fixed points.

6. (a) (i) Using convolution theorem.

Find $L^{-1} \left\{ \frac{(s+2)^2}{(s^2+4s+8)^2} \right\}$

(ii) Evaluate using L.T. $\int_0^{\infty} e^{-t} \left(\frac{\cos 3t - \cos 2t}{t} \right) dt$

(b) Show that $w = \frac{x}{x^2+y^2} - \frac{iy}{x^2+y^2}$ is an analytic function and find $\frac{dw}{dz}$ in terms of z.

(c) Investigate for what value of λ and μ the equations

$$\begin{aligned} 2x + 3y + 5z &= 9 \\ 7x + 3y - 2z &= 8 \\ 2x + 3y + \lambda z &= \mu \end{aligned} \text{ have}$$

- (i) no solution
- (ii) a unique solution
- (iii) an infinite no. of solutions ?

7. (a) Obtain half range cosine series for $f(x) = 1 \quad 0 \leq x \leq 1$
 $= x \quad 1 \leq x \leq 2$

(b) Using Fourier Cosine intergral prove that—

$$e^{-x} \cos x = \frac{2}{\pi} \int_0^{\infty} \frac{(w^2 + 2)}{(w^4 + 4)} \cos wx dw.$$

(c) (i) Show that under the transformation—

$$w = \frac{1}{z} \text{ the circle } |z - 3i| = 3 \text{ is mapped on to the } 6V + 1 = 0.$$

(ii) If $V = 3x^2y + 6xy - y^3$ show that V is harmonic and find the corresponding analytic function.