S.E (Electrical, Electronics, Instrumentation, Bio-Medical and Electronics & Telecommunication) (Sem. III) (Rev.)

Con. 3399-07. May - 2007 ND-2080

16/6/2007 (REVISED COURSE) Applied Mathematics III

(3 Hours) [Total Marks : 100

N.B. (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions out of remaining **six** questions.
(3) Assume any suitable data, wherever **required** but justify the **same**.
(4) **Figures** to the **right** indicate **full** marks.

1. (a) Define Laplace transforms, and If $L\{f(t)\} = f(s)$ and g(t) is a function defined as 5

$$g(t) = \begin{cases} 0, & 0 < t < a \\ f(t-a), & t > a \end{cases}$$ then prove that $L\{g(t)\} = e^{-as} f(s)$

(b) Determine the value of b such that the rank of A is 3 where $A = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 4 & 4 & -3 & 1 \\ b & 2 & 2 & 2 \\ 9 & 9 & b & 3 \end{bmatrix}$ 5

(c) If w = f(z) is analytic then show that $|f'(z)|^2 = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$ 5

(d) Express $f(x) = \frac{1}{2}(\pi - x)$ in a Fourier series with period 2π to be valid in the interval $(0, 2\pi)$. 5

2. (a) Find (1) $L\left\{\left(\sqrt{t} \pm \frac{1}{\sqrt{t}}\right)^3\right\}$ (2) $L\{e^{2t} \sin^4 t\}$ 5

(b) Find the Fourier series of $f(x) = x^2$, $0 < x < 4$ and hence deduce that — 5

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots = \frac{\pi^2}{6}$$

(c) Determine P such that the function $f(z) = \frac{1}{2}\log\left(x^2 + y^2\right) + i \tan^{-1}\frac{px}{y}$ is analytic. 5

(d) Using row transformations find the inverse of the matrix $A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$ 5

3. (a) Using Laplace transforms show that $\int_0^\infty e^{-\sqrt{2}t} \frac{\sinh t \sin t}{t} dt = \frac{\pi}{8}$. 5

(b) Reduce the matrix $A = \begin{bmatrix} 1 & 2 & -2 & 3 & 1 \\ 1 & 3 & -2 & 3 & 0 \\ 2 & 4 & -3 & 6 & 4 \\ 1 & 1 & -1 & 4 & 6 \end{bmatrix}$ 5

to the normal form and hence find its rank.

(c) Find the Fourier series expansion of the function— 5

$$f(x) = \pi x, \quad 0 \le x \le 1$$
$$= \pi (2 - x), \quad 1 \le x \le 2$$

(d) If $f(z) = u + iv$ be analytic function of $z = x + iy$, and $u - v = (x - y)(x^2 + 4xy + y^2)$ 5 then find $f(z)$.

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4. (a) Find non-singular matrix P and Q so that PAQ is a normal form where 5

$$A = \begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

(b) Find (1) $L^{-1}\left\{\frac{s^2}{(s^2+a^2)^2}\right\}$ (2) $L^{-1}\left\{\frac{s+29}{(s+4)(s^2+9)}\right\}$. 5

(c) If $u = \lambda(1 + \cos\theta)$ then find V so that u + iv is analytical. 5

(d) Obtain half range sine series for f(x). 5
where $f(x) = mx, \quad 0 \le x \le \pi/2$

$= m(\pi - x), \quad \frac{\pi}{2} \le x \le \pi$.

5. (a) Show that the transformation $w = \frac{2z+3}{z-4}$ maps the circle $x^2 + y^2 - 4x = 0$ into the s.t. line $4u + 3 = 0$. 5

(b) Use convolution theorem to find $L^{-1}\left\{\frac{(s+2)^2}{(s^2+4s+8)^2}\right\}$. 5

(c) If the matrix $A = \begin{bmatrix} -4 & -3 & -2 \\ -1 & 0 & 1 \\ 2 & 3 & 4 \end{bmatrix}$ then show that adj A is symmetric. 5

(d) Show that the set of functions $e^{-\frac{x}{2}}, e^{-\frac{x}{2}}(1-x), e^{-\frac{x}{2}}(2-4x+x^2)$ are orthogonal over $(0, \infty)$. 5

6. (a) Find the Bilinear transformation which maps the points. 1, i, – 1 of z-plane onto i, 0, – i of w-plane and find the fixed pt's of this transformation. 5

(b) If A is non-singular matrix of order n, prove that — 5
(1) $A(\text{adj } A) = (\text{adj } A)\, A = |A|\, I_n$ (2) $|\text{adj } A| = |A|^{n-1}$

(c) Obtain complex form of Fourier series for $f(x) = \cos h\, 3x + \sin h 3x$ in $(-3, 3)$. 5

(d) Solve $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = e^{-t}\sin t$ with $y(0) = 0$ and $y'(0) = 1$ by Laplace transform method. 5

7. (a) If f(t) is a periodic function of period T, show that $L\{f(t)\} = \frac{1}{1-e^{-ST}}\int_0^T e^{-st} f(t)\, dt$. 5

(b) If u is a regular function, then prove that— $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f'(z)|^2 = 4|f'(z)|^2$. 5

(c) Test the consistency of following system of equation and solve them if possible— 5

$$6x + y + z = -4$$
$$2x - 3y - z = 0$$
$$-x - 7y - 2z = 7.$$

(d) Expand $f(x) = a\left(1 - \frac{x}{l}\right)$ in the range $(0, l)$ in a half range cosine series. 5