S.E. CMPN/I.T. IV 19/6/07
App. math

542-E : 1stHf07

Con. 3344-07. **(REVISED COURSE)** ND-1642

(3 Hours) [Total Marks : 100

N.B. (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** out of remaining **six** questions.
(3) **Figures** to the **right** indicate **full** marks.

1. (a) Prove that the characteristic roots of a Hermition matrix are real. **20**
(b) Solve by Gauss Jordan Reduction method :—

$$x - 2y + 3z = 6$$
$$2x + 3y + 4z = 15$$
$$3x + 2y - 2z = 4$$

(c) State Cauchy's Residue theorem and use it to solve

$$I = \oint_C \frac{z^2}{(z-1)^2 (z-2)} dz$$

Where C is the circle $| Z | = 2{\cdot}5$.

(d) If $y = f(x)$ is a polynomial of 7th degree and $y_0 + y_8 = 734$, $y_1 + y_7 = 524$, $y_2 + y_6 = 374$, $y_3 + y_5 = 282$. Find y_4 assuming $\Delta^8 y = 0$.

2. (a) Find Eigen values and Eigen vectors for the matrix — **6**

$$A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$$

is A diagonalisable ?

(b) Find an iterative formula to determine $\sqrt[3]{N}$ where $(N > 0)$ using Newton Raphson method and hence evaluate $\sqrt[3]{11}$. **6**

(c) Apply Runge-Kutta method of 4th order to find approximate value of y at $x = 1{\cdot}2$ with $h = 0{\cdot}1$ given— **8**

$$\frac{dy}{dx} = x^2 + y^2, \quad y = 1{\cdot}5 \text{ when } x = 1.$$

3. (a) Solve the equations using Gauss Seidal iteration method upto 3 iterations :— **6**

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

(b) Use Lagrange's interpolation formula to find f(4) and interpolating polynomial — **6**

x	0	1	2	5
f(x)	2	3	12	147

(c) Use Residue theorem to evaluate — **8**

(i) $$\int_0^{2\pi} \frac{1}{13 + 5\sin\theta} d\theta$$

(ii) $$\int_{-\infty}^{\infty} \frac{x^2}{(x^2+9)(x^2+4)} dx$$

4. (a) Use Taylor's series method to solve the differential equation — 6

$$\frac{dy}{dx} = 3x + y^2 \text{ with } x_0 = 0, \; y_0 = 1 \text{ at } x = 0{\cdot}1$$

(b) Evaluate $\int_0^{3+i} z^2 \, dz$ along the parabola $x = 3y^2$. 6

(c) (i) Find the Eigen values of adj A and of $A^2 - 2A + I$ 8

where $A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 4 & 2 \\ 0 & 0 & 3 \end{bmatrix}$.

(ii) If $A = \begin{bmatrix} 2 & 3 \\ -3 & -4 \end{bmatrix}$

Prove that $A^{100} = \begin{bmatrix} -299 & -300 \\ 300 & 301 \end{bmatrix}$

5. (a) Show that the matrix $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ is diagonalisable. Find the diagonal form D and transforming matrix p. 6

(b) Obtain Taylor's and Laurent's expansion of $F(z) = \dfrac{z-1}{z^2 - 2z - 3}$ indicating the region of convergence. 8

(c) Evaluate $\int_0^1 \dfrac{1}{1+x^2}\,dx$ between six equal intervals by Simpson's $\frac{1}{3}$rd rule and hence obtain the value of π. 6

6. (a) Verify Cayley Hamilton theorem for the matrix A and hence find A^{-1} and A^4 where 6

$$A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

(b) Using Newton's forward difference interpolation formula to find the no. of students who obtained marks less than 45. 6

Marks	30–40	40–50	50–60	60–70
No. of Students	31	42	51	35

(c) State and prove the Cauchy's integral formula use it to evaluate — 8

$$I = \oint_C \frac{z-1}{(z+1)^2\,(z-2)}\,dz$$

Where C is $|z - i| = 2$.

7. (a) Prove that matix A is derogatory and find its minimal polynomial. 6

Where $A = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$.

(b) (i) Express $f(x) = 2x^3 - x^2 + 3x + 4$ in factorial notation. 6

(ii) With usual notation prove that

$$\Delta = \frac{1}{2}\delta^2 + \delta\sqrt{1 + \frac{1}{4}\delta^2}.$$

(c) (i) Find a root of $\cos x - x e^x = 0$ by Bisection method in four steps. 8

(ii) Find Eigen values and Eigen vectors of $A^3 + I$ where $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$.