

Signals and Systems
 (REVISED COURSE)

Con. 2553-07.

ND-972

(3 Hours)

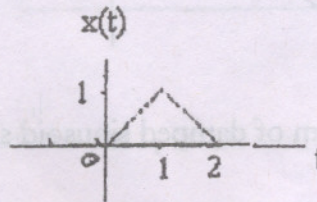
[Total Marks : 100

5 Jan 2007

- N. B. (1) Question No.1 is compulsory
 (2) Attempt any four questions out of remaining six questions.
 (3) Assume suitable data if required

1 (a) Find even and parts of the signal

(20)



(b) Determine whether or not each of the following signals is periodic. If periodic, determine its fundamental period.

(i) $x(t) = \left[\cos\left(2t - \frac{\pi}{3}\right) \right]^2$

(ii) $x(n) = \sin\left(\frac{6\pi}{7}n + 1\right)$

(c) For each of the following input-output relationships, determine whether the corresponding system is linear, time invariant or both.

(i) $y(t) = t^2 x(t-1)$

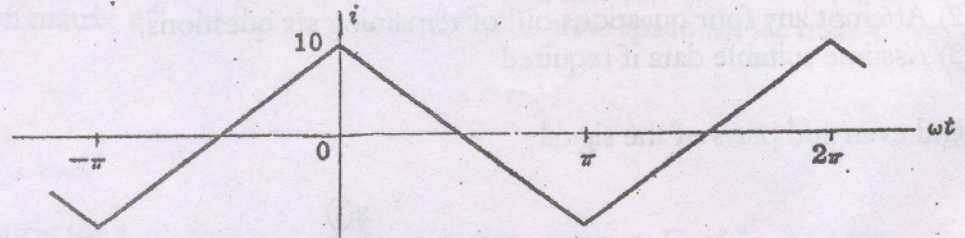
(ii) $y(n) = x(n+1) - x(n-1)$

(d) Show that the necessary and sufficient condition for a relaxed LTI system to be stable under Bounded Input and Bounded Output sense is

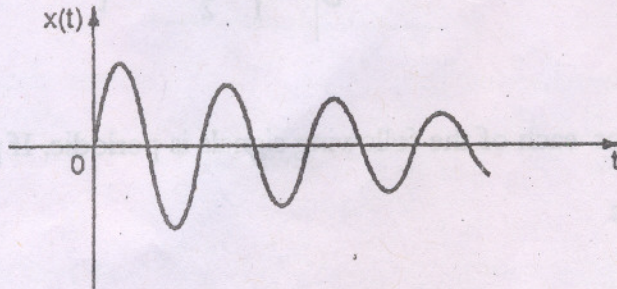
$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

(e) Impulse function has uniform spectral density over the entire frequency interval. Justify.

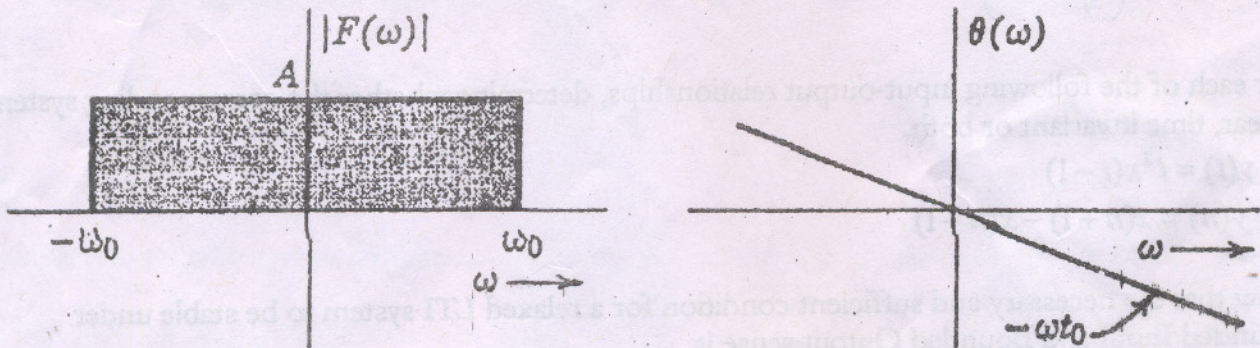
2 (a) The current in an inductance $L = 0.01\text{H}$ has a waveform as shown in Fig. Obtain the trigonometric series for v_L , the voltage across the inductance. $\omega = 500 \text{ rad/s}$. (8)



(b) Find out the Fourier transform of damped sinusoid shown in fig. (6)



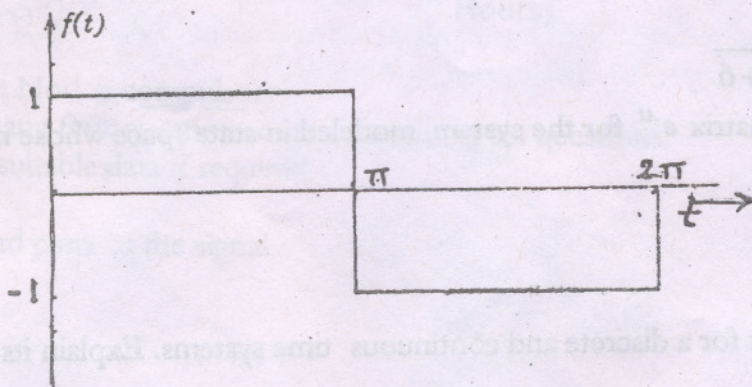
(c) Determine the function $f(t)$ where Fourier Transform is shown in fig. (6)



3 (a) A rectangular function $f(t)$ is defined by

$$f(t) = \begin{cases} 1, & (0 < t < \pi) \\ -1, & (\pi < t < 2\pi) \end{cases}$$

(10)



Approximate this function by a waveform $\sin t$, $\sin 2t$ and $\sin 3t$. Also determine the mean square error.

- (b) Determine the steady state and transient responses of the system characterized by the difference equation $y(n) = 0.5y(n-1) + x(n)$, when the input signal is, $x(n) = 10 \cos(\pi n/4) u(n)$. The system is initially at rest. (10)

4. (a) Convolve $x(n) = \left(\frac{1}{3}\right)^n u(n)$ with $h(n) = \left(\frac{1}{2}\right)^n u(n)$ and verify your answer using z transform. (8)

(b) Derive the relationship between Laplace Transform and Fourier Transform. (6)

(c) Explain the relationship between Discrete Time Fourier Transform and z transform (6)

5. (a) For a LTI discrete time system if the system impulse response is $h(n)$ and the input sequence is $x(n)$ show that the output is given by the discrete time convolution sum as

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k) \quad (6)$$

(b) State and prove the expression for normalized power and normalized energy of continuous time signal $x(t)$. Periodic signals are always power signals. Justify (9)

(c) Prove that for discrete time signal to be periodic, its frequency must be a rational number. What is the minimum and maximum range for discrete time sinusoidal frequency? What is its unit? (5)

6. (a) Find Laplace Transform of $f(t) = e^{-4t}u(-t) + e^{-6t}u(t)$. Does the Laplace Transform exist? Show the ROC. (8)

(b) If $F(s) = \frac{1}{(s+2)(s+3)}$ (8)

(i) Determine the final value by application of final value theorem.

(ii) Verify the results by finding $f(t)$

(c) List properties of Laplace Transform (4)

- 7 a) Obtain the state variable model of the following system.

$$\frac{Y(s)}{U(s)} = \frac{6}{s^3 + 6s^2 + 11s + 6} \quad (8)$$

(b) Find the state transition matrix e^{At} for the system, modeled in state space whose matrix is given by

$$A = \begin{bmatrix} \frac{3}{4} & 0 \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad (8)$$

(c) Define state transition matrix for a discrete and continuous time systems. Explain its significance. (4)