I.E (Computer & Information Technology) (Sem. V) (Rev)

Con. 3333-07. Applied Mathematics V Max 100 ND-1219

(REVISED COURSE)

(3 Hours) [Total Marks : 100

15/5/2007

N.B. (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** out of remaining **six** questions.
(3) **Figures** to the **right** indicate **full** marks.
(4) Use of statistical tables is **permitted**.

1. (a) Three urns contain respectively 3 green and 2 white balls, 5 green and 6 white balls, 2 green and 4 white balls. One ball is drawn from each urn. Find the expected number of white balls drawn out. 5

(b) If the actual amount of instant coffee which the filling machine puts into "6-ounce" jars is a random variable having normal distribution with standard deviation ·05 ounce and if only 3% of jars are to contain less than 6 ounces of coffee, what must be the mean fill of these jars ? 5

(c) For the probability density function 5

$$f(x) = \frac{2(b+x)}{b(a+b)} \qquad -b \le x < 0$$

$$= \frac{2(a-x)}{a(a+b)} \qquad 0 \le x \le a$$

(i) Check that above is a p.d.f.
(ii) Find the mean.

(d) A company produces two types of goods P and Q that require gold and silver. Each unit of type P requires 2 gms of silver and 1 gm of gold, while for Q 1 gm of silver and 2 gms of gold are required. The company has only 90 gms of silver and 80 gms of gold. Each unit of type P gives a profit of Rs. 400 and each unit of type Q gives a profit of Rs. 500. Formulate the L.P.P. and solve it graphically to find the number of units of P and Q the company should produce to maximise the profit. 5

2. (a) For a special security in a certain protected area it was decided to put three lighting bulbs on each pole. If each bulb has probability (p) of burning out in the first 100 hours of service, calculate the probability that atleast one of them is still good after 100 hours, given p = ·3 6

(b) The mean breaking strength of cables supplied by a manufacturer is 1800 with standard deviation 100. By a new technique in the manufacturing process it is claimed that the breaking strength of the cables has increased. In order to test the claim a sample of 50 cables is tested. It is found that the mean breaking strength is 1850. Can we support the claim at 1% level of significance. 6

(c) (i) Explain the term "artificial variable" and its use in linear programming. 2

(ii) Solve using Big-M-methnod the following L.P.P. 6

Minimise $Z = 3x_1 + 8x_2$
Subject to $x_1 + x_2 = 200$
$x_1 \le 80$
$x_2 \ge 60$
$x_1, x_2 \ge 0$

3. (a) An urn contains 4 white and 3 red balls. Find the probability distribution of the number of red balls in three draws made successively with replacement from the urn. 6

(b) Use Simplex method to solve the following L.P.P. 6

Maximise $Z = 5x_1 + 3x_2 + 7x_3$
Subject to $x_1 + x_2 + 2x_3 \le 26$
$3x_1 + 2x_2 + x_3 \le 26$
$x_1 + x_2 + x_3 \le 18$
$x_1, x_2, x_3 \ge 0$

(c) A total number of 3759 individuals were interviewed in a public opinion survey on a political proposal of them 1872 were men and the rest were women. A total of 2257 individuals were in favor of the proposal and 917 were opposed to it. A total of 243 men were undecided and 442 women were opposed to the proposal. Do you justify on contradict the hypothesis that there is no association between sex and attitude, at 5% level of significance. 8

4. (a) If X is binomially distributed with parameter n and p i.e. 6

$$X \sim B(n, p) \quad \text{show that} \quad X \; E\left(\frac{x}{n} - p\right)^2 = \frac{pq}{n}$$

(b) You are given the following data : 6

	X	Y
Arithmetic mean	36	85
Standard deviation	11	08

Correlation coefficient between X and Y = ·66

(i) Find the two regression equations

(ii) Estimate the value of X when Y = 75

(c) Using Kuhn-Tucker conditions solve the following N.L.P.P. 8

Maximise $Z = 2x_1^2 - 7x_2^2 + 12x_1 x_2$

Subject to $2x_1 + 5x_2 \leq 98$

$x_1, x_2 \geq 0$

5. (a) Show that the mean deviation about the mean for a Normal Distribution is $\frac{4}{5}\sigma$ 6

(b) Use Dual Simplex method to solve 6

Maximise $Z = 2x_1 - x_3$

Subject to $x_1 + x_2 - x_3 \geq 5$

$x_1 - 2x_2 + 4x_3 \geq 8$

$x_1, x_2, x_3 \geq 0$

(c) Using the method of Lagrangian multipliers solve the following N.L.P.P. 8

Optimise $Z = 12x_1 + 8x_2 + 6x_3 - x_1^2 - x_2^2 - x_3^2 - 23$

Subject to $x_1 + x_2 + x_3 = 10$

$x_1, x_2, x_3 \geq 0.$

6. (a) (i) Explain (1) Degenerate basic feasible solution 3

(2) Interval estimation and confidence limits

(3) Unbiased estimate

(ii) If M.G.F. (moment generating function) of a discrete random variable X is $e^{4\left(e^t - 1\right)}$ 3

find $P(X = \mu + \sigma)$ where μ and σ are mean and standard deviation of X.

(b) A drug was administred to 5 persons and the systolic blood pressure before and after was measured. 6
The results are given below :

Candidates :	I	II	III	IV	V
B.P. before	140	130	132	150	140
B.P. after	132	126	133	144	133

Test whether the drug is effective in lowering the systolic blood pressure at 5% level of significance.

(c) Write down the Dual of the following L.P.P. and solve it 8

Minimise $Z = 10x_1 + 15x_2 + 30x_3$

Subject to $x_1 + 3x_2 + x_3 \geq 90$

$2x_1 + 5x_2 + 3x_3 \geq 120$

$x_1 + x_2 + x_3 \geq 60$

$x_1, x_2, x_3 \geq 0$

Hence write down the solution of the primal.

7. (a) Find all the basic solutions to the following problem : 6

Maximise $Z = x_1 + 3x_2 + 3x_3$

Subject to $x_1 + 2x_2 + 3x_3 = 4$

$2x_1 + 3x_2 + 5x_3 = 7$

Also find which are basic feasible and non degenerate basic feasible solutions.

(b) Fit a Poisson Distribution to the following data : 6

x	0	1	2	3	4	5	6	7	8
f	56	156	132	92	37	22	4	0	1

(c) Prove that 8

$$\sigma_{x-y}^2 = \sigma_x^2 + \sigma_y^2 - 2r\,\sigma_x\,\sigma_y$$

and using this formula find the coefficient of correlation 'r' between the heights of fathers (x) and of sons (y) from the following data :

x	65	66	67	68	69	70	71	67
y	67	68	64	72	70	67	70	68