

N.B. Answer any five questions.

1. (a) State and explain with example :-
 - (i) Conditional probability 2
 - (ii) Baye's Theorem. 4
- (b) If two events A and B are independent, show that —
 - (i) \bar{A} and \bar{B} are independent 2
 - (ii) A and \bar{B} are independent. 2
- (c) For a certain binary communication channel, the probability that a transmitted '0' is received as a '0' is 0.95 while the probability that a transmitted '1' is received as '1' is 0.90. If the probability of transmitting a '0' is 0.4, find the probability that —
 - (i) a '1' is received 3
 - (ii) a '1' was transmitted given that '1' was received 4
 - (iii) the error has occurred. 3
2. (a) Define probability distribution function of random variable. State important properties of it and prove any one property. 6
- (b) If X is poisson distributed random variable, find moment generating function (M.G.F.) and characteristic function. 6
- (c) A continuous random variable has the probability density function defined by — 8

$$f_x(x) = k \cdot x \cdot e^{-\lambda x} ; \quad x \geq 0, \lambda > 0$$

$$= 0 ; \quad \text{else.}$$

Determine the constant K and find mean and variance.
3. (a) If X is a continuous random variable and $Y = aX + b$ then, prove that — 6

$$f_y(y) = \frac{1}{|a|} f_x\left(\frac{Y-b}{a}\right).$$
- (b) If a random variable X has uniform distribution in $(-2, 2)$, find the probability density function $f_y(y)$ of $Y = 3X + 2$. 4
- (c) The joint probability density function of two dimensional random variable (x, y) is given by —
 - $f_{xy}(x, y) = 4xy e^{-(x^2 + y^2)} ; \quad x \geq 0, y \geq 0$
 - (i) Find the marginal density functions of x and y. 4
 - (ii) Find the conditional density function of Y given that $X = x$ and the conditional density function of X given that $Y = y$. 4
 - (iii) Check for independence of X and Y. 2
4. (a) Prove that if two random variables are independent, then density function of their sum is given by convolution of their density functions. 8
- (b) If X and Y are two independent exponential random variables with probability density functions given by

$$f_x(x) = 2 \cdot e^{-2x} ; \quad x \geq 0$$

$$= 0 ; \quad x < 0 \text{ and}$$

$$f_y(y) = 3 \cdot e^{-3y} ; \quad y \geq 0$$

$$= 0 ; \quad y < 0$$

Find the probability density function of $z = \hat{x} + y$.
- (c) The joint probability density function of (x, y) is $f_{xy}(x, y) = 8 \cdot e^{-(2x + 4y)} ; \quad x, y \geq 0$. 6
 If $U = X/Y$ and $V = Y$, find the joint probability density function of (U, V) and hence find the probability density function of U.
5. (a) If X and Y are two random variables with standard deviations 6_x and 6_y and if C_{xy} is the covariance between them, then prove
 - (i) $C_{xy}(x, y) = R_{xy}(x, y) - E[X] \cdot E[Y]$ 4
 - (ii) $|C_{xy}| \leq 6_x \cdot 6_y$. 4

Also deduce that 2

$$-1 \leq \rho \leq 1.$$

- (b) If $X = \cos \theta$ and $Y = \sin \theta$ where θ is uniformly distributed over $(0, 2\pi)$.
Prove that —
- (i) X and Y are uncorrelated. 5
 - (ii) X and Y are not independent. 5
6. (a) Explain in brief — 12
- (i) WSS process
 - (ii) Poisson process
 - (iii) Queueing system.
- (b) A random process is given by 8
- $$X(t) = \sin(\omega t + y)$$
- where Y is uniformly distributed in $(0, 2\pi)$. Verify whether $X(t)$ is a wide-sense stationary process.
7. (a) Explain power spectral density function. State its important properties and prove any one property. 8
- (b) For a random process having 6
- $$R_{xx}(\tau) = a \cdot e^{-b|\tau|}$$
- find the spectral density function, where a and b are constants.
- (c) The power spectral density of a WSS process is given by — 6
- $$S_{xx}(\omega) = \frac{b}{a} (a - |\omega|) ; |\omega| \leq a$$
- $$= 0 ; |\omega| > a$$
- Find the auto correlation of the function.
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