T.E (I.T) (Sem IV) / Rev 13/6/07

Digital Signal Processing

Con. 3321-07. ND-2020

(REVISED COURSE)

(3 Hours) **[Total Marks : 100**

N.B. (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions out of remaining **six** questions.
(3) Assumptions made should be clearly stated.

1. (a) A causal DT. system has a difference equation $y(n) = x(n) - 0{\cdot}4y\,(n-1) - 0{\cdot}25y\,(n-2)$ What is ROC of this system. **20**

(b) Frequency response of FIR filter is

$$H(e^{jw}) = e^{-3jw}\,(2 + 1{\cdot}8\cos 3w + 1{\cdot}2\cos 2w + 0{\cdot}5\cos w)$$

Find : (i) Impulse Response (ii) Identify filter type based on passband.

(c) Find the energy of the signal

$$x(n) = \left(\frac{1}{2}\right)^n u(n) + 8^n\,u(-n-1)$$

(d) Find Initial and final value of the function :—

$$X(z) = \frac{1+z^{-1}}{1-0{\cdot}25z^{-2}}$$

(e) Determine the range of values of 'a' and 'b', for which LTI system with Impulse response—

$$h(n) = a^n,\ n \geq 0$$
$$= b^n,\ n \leq 0$$

is stable.

2. (a) A causal DT system has transfer function H(z) such that $H(z) = H_1(z)\cdot H_2(z)$.

$H_1(z)$ has one pole at $z = 0 - 5$ and one zero at $z = \frac{1}{3}$

$H_2(z)$ has one pole at $z = 0$ and one zero at $z = \frac{-1}{2}$

(i) Find transfer function of system **3**
(ii) Find Difference equation of system **2**
(iii) Find response of system to i/p — $x[n] = \left(-\frac{1}{2}\right)^n u(n)$ **3**
(iv) Draw pole-zero plot of the overall system and hence comment on the stability of system. **2**

(b) State and prove the convolution theorem of z-transform. Using this property determine the convolution of following pair of signals **10**

$h_1(n) = n\,u\,(n)$
$h_2(n) = 2^n\,u\,(n-1)$

3. (a) If $x(n) = \delta(n) + \delta(n-1) + \delta(n-2)$

(i) Find $X(e^{jw})$ **3**
(ii) Find X(k), 4 – pt DFT **3**
(iii) Show that DFT is sampled version of $|\,x\,(e^{jw})\,|$ **4**

(b) Given that—

$x(n) = \{\,(1 + 2j),\ (1 + j),\ (2 + j),\ (2 + 2j)\,\}$

(i) Find X(k) using DIT-FFT algorithm. **4**
(ii) Using the results in (i) and not otherwise find DFT of p(n) and q(n) where— **6**

$p(n) = \{\,1, 1, 2, 2\,\}$
$q(n) = \{\,2, 1, 1, 2\,\}$

[TURN OVER

4. (a) A low pass filter is to be designed with the following desired frequency response— 10

$$Hd\left(e^{jw}\right) = \begin{cases} e^{-2w} & -\pi/4 \le w \le \pi/4 \\ 0 & \frac{\pi}{4} \le w \le \pi \end{cases}$$

Determine filter coefficients hd(n) if the Window function is defined as

$$w(n) = \begin{cases} 1 & 0 \le n \le 4 \\ 0 & \text{otherwise} \end{cases}$$

Also determine the frequency response $H(e^{jw})$ of the designed filter.

(b) One of the zeros of a third order causal linear phase High Pass FIR filter lies at $z = \frac{1}{2}$. Find the location of other zeros and hence find the transfer function of the filter. 6

(c) Find number of Real additions and Real multiplications required to find DFT for 32 – pt signal. Compare them with number of computations required if FFT algorithm is used. 4

5. (a) Determine the order of filter N and cutoff frequency ΩC for the specification 8

$0{\cdot}8 \le |H(jw)| \le 1 \qquad 0 \le w \le 0{\cdot}2\pi$

$|H(jw)| \le 0{\cdot}2 \qquad 0{\cdot}6\pi \le w \le \pi$ 6

If Bilinear Transformation Technique is used.
Assume T = 1 sec.

(b) Determine H(z) using Impulse Invariant technique for analog system function. 6

$$H(s) = \frac{1}{(s+1)(s+3)}$$

Take sampling period as $T_s = 2$ sec.

(c) Find z transform of $\cos(w_0 n) \cdot u(n)$. 10

6. (a) Impulse Response of FIR filter is given by $h(n) = \{1, \underset{\uparrow}{1}\}$ 10

Find the Response of the system to i/p x(n)
where $x(n) = \{2, \underset{\uparrow}{2}, 4\}$ using FFT/IFFT.

(b) Obtain DF-I, DF-II, cascade and parallel structures for the system described by 10

$$y(n) = -0{\cdot}1\, y(n-1) + 0{\cdot}72y(n-2) + 0{\cdot}7x(n) - 0{\cdot}252\, x(n-2)$$

7. Write short notes on : 20
   (a) Discrete Hilbert Transform
   (b) Bilinear Transformation
   (c) Unilateral z transform.
