

N.B.(1) Question No. 1 is **compulsory**.

(2) Attempt any **four** questions from remaining **six** questions.

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1. (a) Prove that—

$$\int_0^{\infty} x e^{-x^4} dx \times \int_0^{\infty} \frac{e^{-x^2}}{\sqrt{x}} dx = \frac{\pi}{4\sqrt{2}}.$$

(b) Use the rule of D.U.I.S. to prove that—

$$\int_0^{\infty} e^{-\left(x^2 + \frac{a^2}{x^2}\right)} dx = \frac{\sqrt{\pi}}{2} e^{-2a}$$

$$a > 0$$

$$\text{given } \int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}.$$

(c) Solve—

$$\left(y + \frac{1}{3}y^3 + \frac{1}{2}x^2\right) dx + \frac{1}{4}(x + xy^2) dy = 0.$$

(d) Use Taylor's series method to find y at $x = 0.2$ given

$$\frac{dy}{dx} = 1 + y^2 \quad \text{with } y = 0 \quad \text{at } x = 0.$$

2. (a) Solve—

$$(D^3 + D^2 + D + 1) y = \sin^2 x$$

(b) Solve—

$$(D^2 + 2) y = e^x \cos x + x^2 e^{3x}$$

(c) Evaluate $\iint_R \frac{2xy^5}{\sqrt{1+x^2y^2-y^4}} dx dy$

where R is a triangle whose vertices are $(0, 0)$, $(1, 1)$ and $(0, 1)$.

3. (a) Find the mass of Lamina in the form of cardioid $r = a(1 + \cos \theta)$ if the density at any point varies as its distance from the pole of cardioid.

(b) Change the order of Integration $\int_0^a \int_{\sqrt{a^2-x^2}}^{x+3a} f(x,y) dx dy$.

(c) Solve using Runge-Kutta method of order 4.

$$\frac{dy}{dx} = \frac{y}{x} \quad x_0 = 1, \quad y_0 = 1, \quad \text{for } x = 1.2 \quad \text{with } h = 0.1.$$

4. (a) Prove that—

$$\int_0^1 \frac{x^3 - 2x^4 + x^5}{(1+x)^7} dx = \frac{1}{960}.$$

(b) Solve $\frac{dy}{dx} = x + 3y$, $x_0 = 0$ by Euler's modified method for $x = 0.1$ in one step.
 $y_0 = 1$

Compare the answer with exact value.

(c) Solve :

$$(1+x)^2 \frac{d^2y}{dx^2} + (1+x) \frac{dy}{dx} + y = 4 \cos(\log(1+x))$$

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5. (a) Solve—

$$y \, dx + x (1 - 3x^2y^2) \, dy = 0$$

(b) Use polar co-ordinates to evaluate—

$$\iint_R \frac{x^2 + y^2}{x^2 y^2} \, dx \, dy$$

where R is area common to circles

$$x^2 + y^2 = ax \quad \text{and} \quad x^2 + y^2 = by$$

$$a, b > 0.$$

(c) Find total length of astroid curve $x^{2/3} + y^{2/3} = a^{2/3}$.

Also prove that the line $\theta = \frac{\pi}{3}$ divides the arc of astroid in +ve quadrant in the ratio 1:3.

6. (a) Evaluate—

$$\iiint_V x^2 \, dx \, dy \, dz \quad \text{over volume of tetrahedron bounded by}$$

$$x = 0, \quad y = 0, \quad z = 0, \quad \text{and} \quad \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1.$$

(b) Change the order of integration and evaluate.

$$\int_0^2 \int_{\sqrt{2y}}^2 \frac{x^2 \, dx \, dy}{\sqrt{x^4 - 4y^2}}$$

(c) Solve by the method of variation of parameters.

$$(D^2 - 6D + 9) y = \frac{e^{3x}}{x^2}.$$

7. (a) Find the volume of solid bounded by cylinder

$$x^2 + y^2 = 2ay, \quad \text{the paraboloid} \quad x^2 + y^2 = az \quad \text{and the plane} \quad z = 0.$$

(b) The radial displacement in a rotating disc at a distance r from the axis is given by

$$\frac{d^2y}{dr^2} + \frac{1}{r} \frac{du}{dr} \frac{u}{r^2} + Kr = 0$$

Find the displacement if $u = 0$ at $r = 0$ and at $r = a$.

(c) Prove that—

$$(i) \quad \beta(m, m) \times \beta\left(m + \frac{1}{2}, m + \frac{1}{2}\right) = \frac{\pi}{m} 2^{1-4m}$$

$$(ii) \quad \int_0^\infty \frac{x^4 (1+x^5)}{(1+x)^{15}} \, dx = \frac{1}{5005}.$$