

- N.B.** (1) Question No. 1 is **compulsory**.
 (2) Attempt any **four** questions out of remaining **six** questions.
 (3) Assume **suitable** data if **necessary**.
 (4) **Figures** to the **right** indicate **full marks**.

1. (a) Obtain the complex form of Fourier series for $f(x) = \cos h ax$ in $(-\pi, \pi)$ where a is not an integer. 5

(b) Show that $w = \frac{x}{x^2 + y^2} - \frac{iy}{x^2 + y^2}$ is an analytic function and find $\frac{dw}{dz}$. 5

(c) Prove that— $\int_0^\infty e^{-t} \frac{\sin^2 t}{t} dt = \frac{1}{4} \log 5$. 5

(d) Express the matrix : 5

$$A = \begin{bmatrix} 2 & 4+i & 4i \\ 3i & 6-i & 2 \\ 6 & 4-2i & 1-i \end{bmatrix}$$

as the sum of a Hermitian matrix and Skew Hermitian matrix.

2. (a) Find Fourier Series for 6

$$f(x) = \frac{1}{2} (\pi - x) \text{ in } (0, 2\pi) \text{ hence deduce that } \frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \dots$$

(b) Find (i) $L \{ t e^{-3t} \cos 2t \}$ 3

(ii) $L \left\{ e^{-3t} \int_0^t u \sin 3u \right\}$ 4

(c) Use Laplace Transform to solve— 7
 $(D^2 - 3D + 2) y = 4t + e^{3t}$ if $y(0) = 1, y'(0) = -1$

3. (a) Find the inverses of $\begin{bmatrix} 1 & a & 0 \\ 0 & 1 & a \\ 0 & 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 & 0 \\ b & 1 & 0 \\ 0 & b & 1 \end{bmatrix}$ 6

hence find the inverse of $\begin{bmatrix} 1+ab & a & 0 \\ b & 1+ab & a \\ 0 & b & 1 \end{bmatrix}$

(b) If A and B are given below. Find the rank of A by reducing it to normal form. Find $3A - B$. Hence or otherwise show that $3A^2 - AB = 2A$, also find the rank of $3A^2 - AB$. 6

where $A = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 2 & 1 & 1 \\ 2 & 6 & 3 & 5 \\ 2 & 4 & 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 6 & 3 & 6 \\ 0 & 4 & 3 & 3 \\ 6 & 18 & 7 & 15 \\ 6 & 12 & 6 & 10 \end{bmatrix}$

(c) Find (i) $L^{-1} \left\{ \frac{s^2 + 2s + 3}{(s^2 + 2s + 5)(s^2 + 2s + 2)} \right\}$ 8

(ii) $L^{-1} \left\{ \tan^{-1} \frac{2}{s^2} \right\}$

4. (a) Show that the set of functions $\sin (2n + 1) x$, $n = 0, 1, 2 \dots$ is orthogonal over $[0, \pi/2]$. Hence construct the orthonormal set of functions. 6

(b) Find Fourier series for $f(x) = | x |$ in $(- \pi, \pi)$ hence deduce that 6

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} \dots$$

(c) Find an analytic function whose real part is $u = e^{2x} (x \cos 2y - y \sin 2y)$ also verify that V is an harmonic function. 8

5. (a) If $A = \frac{1}{3} \begin{bmatrix} 1 & 2 & a \\ 2 & 1 & b \\ 2 & -2 & c \end{bmatrix}$ is orthogonal. Find a, b, c , also find rank A 6

(b) Using Convolution theorem Evaluate $L^{-1} \left\{ \frac{1}{(s^2 + 4s + 13)^2} \right\}$ 6

(c) Find the bilinear transformation which maps the points $z = 1, i - 1$ on to the points $w = i, 0, -i$ hence find fixed points of the transformation. 8

6. (a) Using Laplace transform to evaluate 6

$$\int_0^{\infty} e^{-t} (1+2t -3t^2 + 4t^3) H(t - 2) dt.$$

(b) Show that $u = \frac{1}{2} \log (x^2 + y^2)$ satisfies the Laplace equation find its corresponding analytic function and the harmonic conjugate. 6

(c) For what value of λ the equations $x + y + z = 1, x + 2y + 4z = \lambda, x + 4y + 10z = \lambda^2$ have a solution and solve them completely in each case. 8

7. (a) Find two non singular matrices P and Q such that PAQ is in normal form : 4

where $A = \begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$

(b) Obtain half range sine series for $f(x) = x (2 - x)$ in $0 < x < 2$ and hence deduce that 8

$$\sum_{n=1}^{\infty} \frac{1}{n^6} + \frac{\pi^6}{945}$$

(c) (i) Show that under the transformation $w = \frac{1}{z}$ the circle $| z - 3i | = 3$ is mapped onto the line $6u + 1 = 0$. 4

(ii) Prove that the curves $r^n = a \sec n \theta$ and $r^n = b \operatorname{cosec} n \theta$ intersect orthogonally . 4