S.E. Sem 3 (Rev:) Discrete Structures. 04/06/08
ihf-23-5-nk-08. 72
Comp & I.T.

Con. 3107-08. **CO-9565**

(REVISED COURSE)

(3 Hours) **[Total Marks : 100**

N.B.(1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions out of remaining **six** questions.
(3) Assumption made should be clearly stated.
(4) **Figures** to the **right** indicate **full** marks.

1. (a) Prove that if $A \subseteq C$ and $B \subseteq C$, then $A \cup B \subseteq C$. 4
(b) Suppose that $A \oplus B = A \oplus C$. Does this guarantee that B = C ? Justify your conclusion. 4
(c) Let A = { x I x is a real number and $0 < x < 1$ }, B = { x I x is a real number and $x^2 + 1 = 0$ }, C = { x I x = 4 m, $m \in z$ }, D = { (x, 3) I x is an english word, whose length is 3 }, and E = { x I $x \in z$ and $x^2 \leq 100$ }. Identify each set as finite, countable, or uncountable. 6
(d) Suppose that 109 of the 150 computer science students at one of the Mumbai college take at least one of the following computer language :— VB, VC ++ and Java. Suppose 45 study VB, 61 study V C + +, 53 study Java, 18 Study VB and VC ++, 53 study VC + + and Java, and 23 study VB and Java. 6
(i) How many students study all three languages ?
(ii) How many students study only VC ++ ?
(iii) How many students do not study any of the language ?

2. (a) Prove that if x is a rational number and y is an irrational number, then x + y is an irrational number. 6
(b) Let P(n) be the statement $n^2 + n$ is an odd number for $n \in z^+$ 6
(i) Prove that $P(k) \Rightarrow P(k + 1)$ is a tautology.
(i) IS P(n) true for all n ? Explain.
(c) Prove that if any 14 integers from 1 to 25 are Chosen, then one of them is a multiple of another. 4
(d) Solve the recurrence relation $d_n = 2d_{n-1} - d_{n-2}$ with initial conditions $d_1 = 1{\cdot}5$ and $d_2 = 3$. 4

3. (a) Let A = R × R. Define the following relation R on A : $(a_1\ b)$ R (c, d) if and only if $a^2 + b^2 = c^2 + d^2$. 6
(i) Show that R is an equivalent relation.
(ii) Compute A/R.
(b) Let R be a relation on a set a that has n elements. Show that $MR^* = MR^\infty$ VIn. Where In is the n × n identify matrix. 6
(c) Let R be a nonempty relation on a set A. Suppose that R is symmetric and transitive. Show that R is not irreflexive. 4
(d) If { { a, c, e }, { a, d, f } } is a partition of the set A = { a, b, c, d, e, f }, determine the corresponding equivalence relation R. 4

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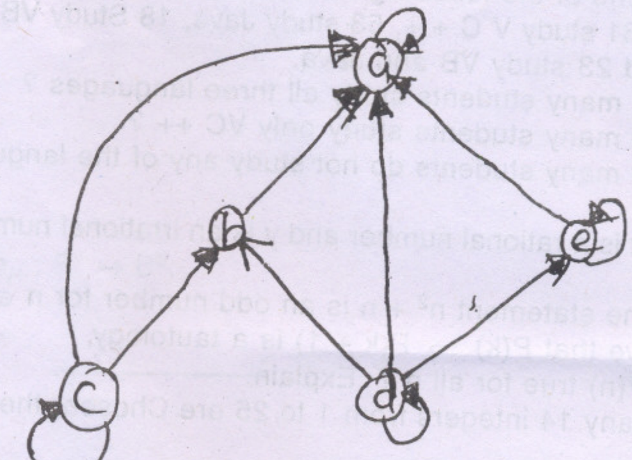
4. (a) Let f : A → B be a function with finite domain and Range. Suppose that I Dom(f) I = n and I Ran (f) I = m. 6
Prove that—
(i) If f is one to one, then m = n.
(ii) If f is not one to one then m < n.

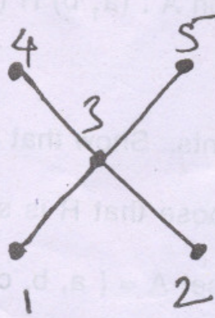
(b) Assume that 9,500 account records need to be stored using the hashing function h, which takes the first two digits of the account number as one number and last four digits as another number, adds them, and then applies the mod-89 Function. 6
(i) How many linked lists will be needed.
(ii) If an approximately even distribution of records is achieved, roughly how many records will be stored in each linked list ?
(iii) Compute h(473810), h(125332), and h(308691).

(c) Determine the Hasse diagrams of the : 4
(i) Relation R, A = { 1, 2, 3, 4 },
R = { (1, 1), (1, 2), (2, 2), (2, 4), (1, 3), (3, 3), (3, 4), (1, 4), (4, 4) }
(ii) Partial order having the given diagraph.



(d) Determine all maximal and minimal elements of the poset. 4
(i) A = { x I x is a real number and 0 < x $\leq$ 1 } with the usual partial order $\leq$.

(ii)



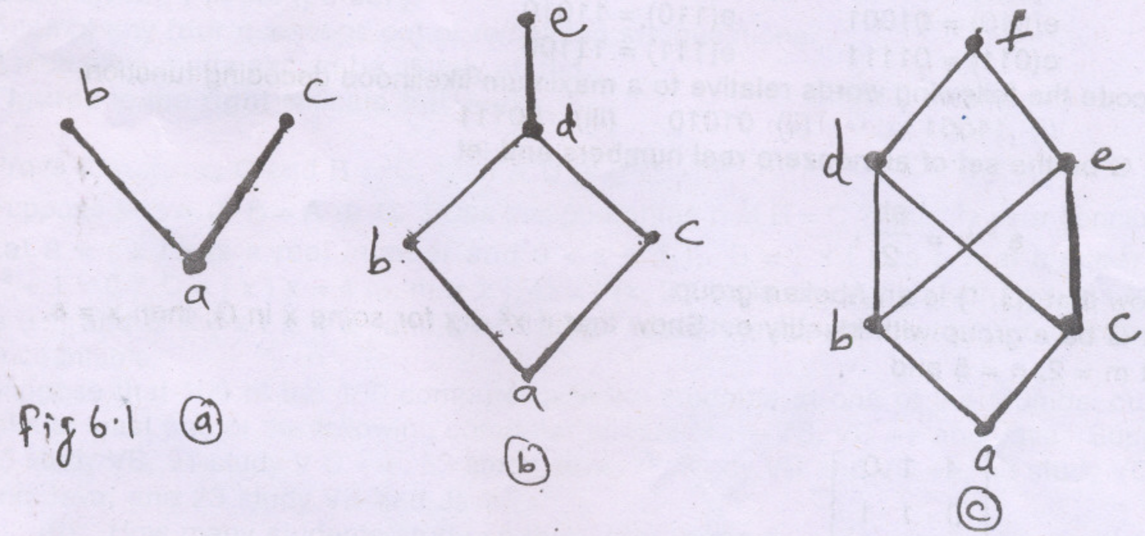
5. (a) Let A { 1, 2, 3, 5, 6, 10, 15, 30 } and consider the partial order $\leq$ of divisibility on A. That is, define a $\leq$ b to mean that alb. Let A' = P(s), where s = { e, f, g }, be the poset with partial order $\subseteq$. Show that (A, $\leq$) and (A', $\subseteq$) are isomorphic 6

(b) Give the Hasse diagrams of all nonisomorphic lattices that have one, two, three, four, or five element. 6

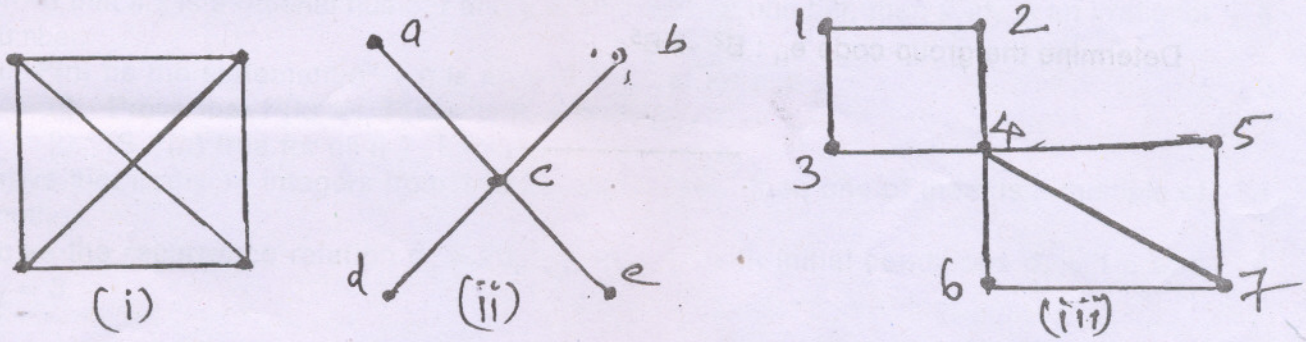
(c) Let L be a distributive lattice. Show that if there exists an a with a ^ x = a ^ y and a v x = a v y, then x = y. 4

(d) Show that a linearly ordered poset is a distributive lattice. 4

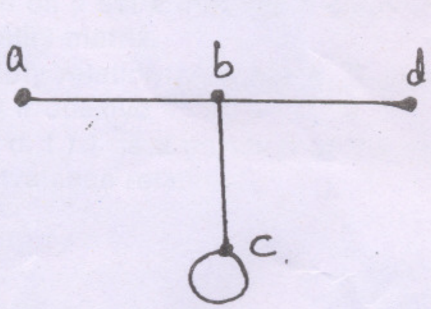
6. (a) Consider the Hasse diagrams given in **figure** 6·1 (a), (b), (c). **6**
   (i) Which of these posets are not lattices ? Explain.
   (ii) Which of these posets are not Boolean algebras ? Explain.



(b) Determine whether the graph shown has a Hamiltonian circuit, a Hamiltonian path but no Hamiltonian circuit, or neither. If the graph has a Hamiltonian circuit, give the circuit. **6**



(c) Show that (D (105), g.c.d., l.c.m.) is a uniquely complemented lattice. **4**
(d) Give the degree of each vertex in **figure** **4**



7. (a) Consider the (3, 5) group encoding Function $e : B^3 \to B^5$ defined by— 6

e(000) = 00000	e(100) = 10011
e(001) = 00110	e(101) = 10101
e(010) = 01001	e(110) = 11010
e(011) = 01111	e(111) = 11100

Decode the following words relative to a maximum likelihood decoding function.

(i) 11001 (ii) 01010 (iii) 00111

(b) Let G be the set of all nonzero real numbers and let 6

$$a * b = \frac{ab}{2}.$$

Show that (G, *) is an Abelian group.

(c) Let G be a group with identify e. Show that if $x^2 = x$ for some x in G, then x = e. 4

(d) Let m = 2, n = 5 and 4

$$H = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Determine the group code $e_H : B^2 \to B^5$.