Con. 3528–08. [REVISED COURSE] CO–9523

(3 Hours) [Total Marks : 100

N.B. : (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions from remaining **six** questions.
(3) **Figures** to the **right** indicate **full** marks.

1. (a) State and prove Parseval's identity over $(c, c + 2l)$. 5
   (b) Find orthogonal trajectories of the family of curves $e^{-x} \cos y + xy = \text{const.}$ 5
   (c) Find $L^{-1}\left\{\dfrac{2}{(s+1)^2\,(s^2+4)}\right\}$. 5
   (d) S.T. every square matrix can be uniquely expressed as a sum of Hermitian and Skew–Hermitian matrix. 5

2. (a) Solve using Laplace transform – 6

$$\frac{dy}{dt} + 2y + \int_0^t y\,dt = \sin t. \text{ given } y(0) = 1.$$

   (b) Find analytic function f(z) whose real part is $\dfrac{\sin 2x}{\cosh 2y + \cos 2x}$. 6
   (c) (i) If 'A' is a nonsingular square matrix of order 'n' then S.T. 4

$$\text{adj}\cdot(\text{adj}\cdot A) = |A|^{n-2}\cdot A.$$

   (ii) Verify $A(\text{adj}\cdot A) = |A|\, I$ 4

$$\text{Where } A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix}.$$

3. (a) Use Laplace transform to evaluate – 6

$$\int_0^{\infty}\int_0^{t} e^{-t}\,\frac{\sin u}{u}\,du\,dt.$$

   (b) S.T. every bilinear transformation is the resultant of three basic transformations. 6
   (c) Find Fourier series for – 8

$$f(x) = \left(\frac{\pi - x}{2}\right)^2 \quad \text{over } (0, 2\pi).$$

Hence S.T. (i) $$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} \,\ldots\ldots\ldots\ldots$$

(ii) $$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \,\ldots\ldots\ldots\ldots$$

4. (a) Find A^{-1} by using elementary transformations – 6

$$\text{Where } A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}.$$

(b) Find Fourier series for – 6

$f(x) = x\cos x \; ; \; -\pi < x < \pi.$

(c) S.T. the relation $W = \dfrac{iz+2}{4z+i}$ transforms the real axis in z–plane into a circle in w–plane. 8

Find its centre and radius. Also find the point in z–plane which is mapped on the centre of the circle in w–plane.

5. (a) For what values of λ and μ the system 6

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

has (i) no solution (ii) unique solution (iii) more than one solutions. Also find parametric solution.

(b) Find half range Cosine series for 6

$$f(x) = \begin{cases} 1 & 0 < x < 1 \\ x & 1 < x < 2 \end{cases}$$

(c) Find (i) $L^{-1}\left\{\dfrac{e^{-\pi s}}{s^2 - 2s - 2}\right\}$ 4

(ii) $L^{-1}\left\{\dfrac{1}{\left(s^2 - q^2\right)^2}\right\}$ 4

6 (a) If $f(t) = \begin{cases} 1 & 0 \le t \le 1 \\ 0 & 1 < t < 2 \end{cases}$ 6

and $f(t) = f(t+2)$ then

S. T. $L\{f(t)\} = \dfrac{1}{s\left(1+e^{-s}\right)}.$

(b) Define orthogonal and orthonormal set of functions. S.T. $\{\sin nx\}_{n=1,2,3,\ldots\ldots}$ is orthogonal set of functions over $[0, \pi]$. Hence construct orthonormal set of functions. 6

(c) Find Fourier series for 8

$f(x) = 2x - x^2 \; ; \; 0 \le x \le 2.$

7. (a) Find complex form of Fourier series for – 6

$f(x) = e^{ax} \; ; \; (-L, L).$

(b) Find (i) $L\left\{\dfrac{\sqrt{1+\sin 4t}}{e^{2t}}\right\}$ 3

(ii) $L\{(t\sin 2t)^2\}.$ 3

(c) State convolution theorem and use it to find $L^{-1}\left\{\dfrac{s}{\left(s^2+4\right)^2}\right\}.$ 8

OR

(c) Define cross ratio. Find Bilinear transformation which transforms point $z = i, 1, -1$ into $w = 1, 0, \infty$ respectively. 8