S.E. Sem 4 (Rev.) Comp & I.T. Applied Maths IV 19/06/08

Con. 3511-08. **CO-9763**

(REVISED COURSE)

(3 Hours) **[Total Marks : 100**

N.B. (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions out of remaining **six** questions.
(3) **Figures** to the **right** indicate **full** marks.

1. (a) Evaluate $\int_C (2 - z^2)\, dz$ where C is the upper half of the circle $|z - 2| = 3$. 5
(b) Solve the equations by using Gauss Jordan reduction method 5
$2x + 3y + 4z = 20, \quad 4x + 3y + 2z = 16, \quad x + 2y + z = 8.$
(c) Prove that the characteristic roots of Hermition matrix are real. 5
(d) If $y = f(x)$ is a polynomial of 7^{th} degree and $y_0 + y_8 = 223$, $y_1 + y_7 = 163$, $y_2 + y_6 = 128$, $y_3 + y_5 = 123$. Find y_4 assuming $\Delta^8 y = 0$. 5

2. (a) Find the eigen values and eigen vectors of the matrix A, if $A = \begin{bmatrix} 7 & -2 & 1 \\ -2 & 10 & -2 \\ 1 & -2 & 7 \end{bmatrix}$ 6
(b) State Cauchy's Residue theorem use it to Evaluate $\oint_C \frac{z-1}{(z+1)^2 (z-2)}\, dz$ where C is the circle $|z - i| = 2$. 6
(c) Using Runge Kutta method or Fourth order to find the approximate value of— 8
$$\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}, \quad \text{given } y(0) = 1, \text{ at } x = 0{\cdot}2 \text{ and } x = 0{\cdot}4.$$

3. (a) Using Lagranges interpolation formula to find the interpolating polynomial f(x) and f(3). 6

x	0	1	2	5
f(x)	2	3	12	147

(b) Test whether the matrix A. $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$ is derogatory and find minimal polynomial. 6
(c) Use Residue theorem to evaluate 8
(i) $\int_0^{2\pi} \frac{1}{5 + 4\cos\theta}\, d\theta$
(ii) $\int_0^{\infty} \frac{dx}{(x^2 + a^2)^2} \quad a > 0$.

4. (a) Find the root of $x^4 - x - 10 = 0$ correct to three places of decimals using Newton Raphson method. 6

(b) Find the symmetric matrix $A_{3 \times 3}$ having the eigen values $\lambda_1 = 0$, $\lambda_2 = 3$ and $\lambda_3 = 15$ with the corresponding eigen vectors. 6

$x_1 = [1, 2, 2]'$, $x_2 = [-2, -1, 2]'$ and x_3.

(c) (i) If $y = a\, 3^x + b(-2)^x$ and $h = 1$ 4

Prove that $(\Delta^2 + \Delta - 6)\, y = 0$

(ii) Express f(x) into factorial polynomial when $F(x) = 2x^3 - 3x^2 + 5x - 4$ and find the function whose first difference is given function. 4

5. (a) Solve the equations by using Gauss Seidal method upto two iterations 6

$43x + 2y + 32 = 91$, $3x + 53y + 2 = 60$, $2x - 4y + 49z = 49$.

(b) Verify Cayley Hamilton theorem for the matrix 6

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$$ and hence find A^{-1}.

(c) Expand $f(z) = \dfrac{(z-2)(z+2)}{(z+1)(z+4)}$ in the region 8

(i) $|z| < 1$ (ii) $1 < |z| < 4$ (iii) $|z| > 4$.

6. (a) Evaluate— 6

$$\int_0^1 \frac{1}{1+x^2}\, dx$$

between six equal intervals and hence find an approximate value of π by Simpson's $\frac{1}{3}$rd rule.

(b) If $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ 6

Find eigen values of A also find eigen value of $4A^{-1}$ and eigen vector of $A^2 - 4I$.

(c) State and prove Cauchy's integral formula use it to evaluate 8

$$\oint_C \frac{z+4}{z^2 + 2z + 5}\, dz$$ where C is circle $|z + 1 + i| = 2$.

7. (a) Use Taylor's Series method to find the approximate value of y at x = 4·1. Given 6

$\dfrac{dy}{dx} = \dfrac{1}{x^2 + y^2}$, $x_0 = 4$, $y_0 = 4$.

(b) Use Newton's Backward interpolation formula to estimate the profit in the year 1925. 6

Year	**1891**	**1901**	**1911**	**1921**	**1931**
Profit in Lakhs	46	66	81	93	101

(c) (i) Is the matrix A diagonisable ? Justify your answer : 4

$$A = \begin{bmatrix} 1 & -2 & 0 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

(ii) Determine the pole of function $f(z) = \dfrac{z^2}{(z-1)^2 (z+2)}$ and also find the Residue at each pole. 4