

N.B. : (1) Question No. 1 is **compulsory**.

(2) Answer any **four** questions out of remaining **six** questions.

1. (a) A and B toss a fair coin alternately one who gets a head first wins Rs. 12. A starts. Find their expectation. 5
- (b) Consider an equilateral triangle whose sides are of length 3 units. If ten points are chosen lying on or inside the triangle, then show that at least two of them are no more than 1 unit apart. 5
- (c) Let A be a set of non-zero integers and let R be relation defined by $(a, b) R (c, d)$ if $ad = bc$. Prove that R is an equivalence relation. 5
- (d) In a binomial distribution consisting of 5 independent trials, probabilities of one and two successes are 0.4096 and 0.2048 respectively. Find the probability of getting 3 success and also determine the mean of the distribution. 5
2. (a) A certain drug is claimed to be effective in curing cold in an experiment on 500 persons with cold. Half of them were given drug and half of them were given the sugarpills. The patients reaction to the treatment are recorded in the following table :- 8

	Helped	Harmed	No effect	Total
Drug	150	30	70	250
Sugarpills	130	40	80	250
Total	280	70	150	500

On the basis of this data, can it be concluded that the drug and sugarpills differ significantly in curing cold.

- (b) A continuous random variable x has a p.d.f. $f(x) = kx^2 e^{-x}$, $x \geq 0$. Find k , mean and variance and m.g.f. about origin for the distribution. 6
- (c) Obtain the line of regression for the following data between two variable x and y : 6

x	6	2	10	4	8
y	9	11	5	8	7

3. (a) If $R(t) = \log M(t)$, where $M(t)$ is the m.g.f. about the origin of a discrete random variable then prove that $\mu = R'(0)$ and $\sigma^2 = R''(0)$. Hence, find the mean and variance of the Poisson distribution. And find $P(\mu - 2\sigma < x < \mu + 2\sigma)$ if $\mu = 4$. 8
- (b) Check whether $A = \{2, 4, 12, 16\}$ and $B = \{3, 4, 12, 24\}$ are Lattices under divisibility. Draw their Hasse diagrams. 6
- (c) Is the following function : (i) Injective (ii) Surjective $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = 2x^2 + 5x - 3$. 6
4. (a) The probability that the marks of a student chosen at random will exceed 50 (out of 100) is 0.25. Find in two different ways, the probability that out of 100 students of this group 25 to 30 will have marks more than 25. 8
- (b) Test the significance of the difference between the means of the samples, drawn from two normal populations with same standard deviation using following data : 6

	Size	Mean	S.D.
Sample 1	100	61	4
Sample 2	200	63	6

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- (c) Calculate the coefficient of correlation for the following data : 6

x	1	2	3	4	5	6	7	8	9
y	9	8	10	12	11	13	14	16	15

5. (a) (i) Define Isomorphic Lattice 8

Let $A = \{2, 4, 8, 12, 36\}$ and $B = \{3, 6, 9, 12, 24\}$ and Let \leq be the relation of divisibility. Are the lattices isomorphic ?

- (ii) Define complemented Lattice. Show that L is not distributive
 $L = \{1, 3, 5, 9, 15, 45\}$ with the relation "is divisible by".

- (b) The following data relates to the marks obtained by 11 students in two tests, one held at the beginning of the year and the other at the end of the year after giving intensive coaching : 6

Test 1	19	23	16	24	17	18	20	18	21	19	20
Test 2	17	24	20	24	20	22	20	20	18	22	18

Do the data indicate the students are benefited by coaching ?

- (c) If f and g are defined as 6

$f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = 2x - 3$

$g : \mathbb{R} \rightarrow \mathbb{R}, g(x) = 4 - 3x$

Find $f \circ g, f^{-1}, g^{-1}$ and verify that $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

6. (a) Prove that $(\mathbb{Z}_5, +_5, \times_5)$ is a ring. Is it an integral domain ? Is it a field ? 8

- (b) Write short note on : 6

(i) Null Hypothesis and alternative Hypothesis

(ii) Type I and Type II error.

(iii) Degree of freedom.

- (c) Data was collected over a period of 10 years, showing number of deaths from horse-kicks in each of 200 army corps. The distribution of the death was as follows : 6

No. of death	0	1	2	3	4	Total
Frequency	109	65	22	3	1	200

Fit a Poisson distribution to the data and calculate the theoretical frequencies.

7. (a) Show that the set $F = \{a + b\sqrt{2}\}$, where a and b are rational numbers is a field under addition and multiplication. 8

- (b) Fit a second degree parabolic curve to the following data by the method of least squares and estimate the production in 1982. 6

Year (x)	1974	1975	1976	1977	1978	1979	1980	1981
Production (y) in tons	12	14	26	42	40	50	52	53

- (c) If R is the set of all real numbers other than zero and if $a * b = 2ab$. Prove that $(R, *)$ is an abelian group. 6