T.E. Sem 6 (Rev.) Probability and Random Processes 16/6/08
EXTC

P4/RT-Ex-08-704

Con. 3496–08. **[REVISED COURSE]** **CO–2944**

(3 Hours) **[Total Marks : 100**

N.B. : Answer any **five** questions.

1. (a) Give the following definitions of probability with the short comings if any :– **8**
   (i) A – priori or classical definition
   (ii) A – posteriori or relative frequency definition
   (iii) Axiomatic definition.

   (b) State and prove Bayes' theorem. **4**

   (c) In a factory, four machines A_1, A_2, A_3 and A_4 produce 10%, 20%, 30% and 40% of the items respectively. The percentage of defective items produced by them is 5%, 4%, 3% and 2% respectively. An item selected at random is found to be defective. What is the probability that it was produced by the machine A_2 ? **8**

2. (a) Define discrete and continuous random variables. Give one example of each type. Define Expectation of discrete random variable and continuous random variable. **10**

   (b) The joint density function of two continuous random variables is given by **10**

$$f(x, y) = \begin{cases} xy/8 & 0 < x < 2, \quad 1 < y < 3 \\ 0 & \text{otherwise} \end{cases}$$

   Find : (a) $E(X)$
   (b) $E(Y)$
   (c) $E(2X + 3Y)$.

3. (a) Suppose X and Y are two random variables. Define covariance and correlation coefficient of X and Y. When do we say that X and Y are – **10**
   (i) Orthogonal
   (ii) Independent and
   (iii) Uncorrelated

   Are uncorrelated random variables independent ?

   (b) Prove $-|C_{xy}| \le \sigma_x \cdot \sigma_y$. **10**
   If x, y are two random variates with standard deviations σ_x and σ_y and if C_{xy} is the covariance between them.

4. (a) Define Entropy of a Discrete Random Variable. **2+8**

   (b) A random sample X has the following probability mass function.

X = x	1	2	3	4	5	6
P(X = x)	3/8	3/8	1/8	1/16	1/32	1/32

   Find the Entropy.

(c) The joint probability function of two random variables x an y is given by :–

$$f(x, y) = \begin{cases} c\left(x^2 + 2y\right) & x = 0, 1, 2 \\ 0 \text{ otherwise} & y = 1, 2, 3, 4 \end{cases}$$

Find (a) the value of c,
(b) $p(x = 2, y = 3)$
(c) $p(x \leq 1, y > 2)$ and
(d) marginal probability functions of x and y.

5. (a) Let $f_{xy}(x,y) = 1, \quad 0 < |y| < x < 1$ **10**
$= 0 \quad$ otherwise.
Determine E(X/Y) and E (Y/X)

(b) Define a random process giving an example. **10**
Define (i) mean
(ii) autocorrelation and
(iii) autocovariance of a random process.

6. (a) If X(t) is an ergodic process, show that $S_{xx}(w) = \int_{-\infty}^{\infty} R_{xx}(\tau)e^{-jwt}\, d\tau$ where **10**
$\tau = t_2 - t_1$, t_1 and t_2 being two instants of time.

(b) Explain power spectral density function. State its important properties and prove any one property. **10**

7. (a) Explain in brief :– **10**
(i) WSS process
(ii) Poisson process
(iii) Queueing system.

(b) If $x = \cos\theta$ and $y = \sin\theta$ where θ is uniformly distributed over $(0, 2\pi)$. **10**
Prove that –
(i) x and y are uncorrelated
(ii) x and y are not independent.
