Con. 2906–09. BB–5664

(3 Hours) [Total Marks : 100

N.B. : (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions out of remaining **six** questions.
(3) Assume **suitable** data if **necessary**.
(4) **Figures** to the **right** indicate **full** marks.

1. (a) Determine the energy of the signal $x(n) = \begin{cases} \left(\frac{1}{2}\right)^n & n \geq 0 \\ (4)^n & n < 0 \end{cases}$ 5

(b) If $H(z) = \frac{2(z-2)}{z-0.5}$, find passband of the filer. 5

(c) Derive a relation between auto correlation of input, impulse response of the system and autocorrelation of output. 5

(d) Give the transfer function H(s) and show pole locations of third order low pass normalised analog Butterwort filter. 5

2. (a) A linear times invarient system is described by the following difference equation : 10
$y(n) = ay(n-1) + bx(n), \quad 0 < a < 1$

(i) Determine the magnitude and phase of the frequency response H(w) of the system.

(ii) Choose the parameter b so that the maximum value of $|H(w)|$ is unity, and sketch $|H(w)|$ and $\angle H(w)$ for a = 0.9

(iii) Determine the output of the system to the input signal

$$x(n) = 5 + 12 \sin \frac{\pi}{2} n - 20 \cos\left(\pi n + \frac{\pi}{4}\right)$$

(b) Consider the analog signal 5
$x_a(t) = 3 \cos 100\pi t$

(i) If the signal is sampled at the rate of $F_s = 75$ HZ, what is the discrete time signal obtained after sampling ?

(ii) What is the frequency $0 < F < \frac{F_s}{2}$ of a sinusoid that yields samples identical to those obtained in part (i).

(c) Determine the convolution of the following pairs of signals : 5

$$x_1(n) = \left(\frac{1}{2}\right)^n u(n)$$

$$x_2(n) = \cos\pi n \; u(n)$$

3. (a) A system has impulse response $h(n) = (0.7)^n\, u(n) - (0.5)^n\, u(n)$ 5
Show Direct form II and parallel realization of the system.

(b) A system is describe by the difference equation $y(n) = \frac{1}{2}\, y(n-1) + x(n)$. 10

Input given to the system is $x(n) = \left(\frac{1}{3}\right)^n\, u(n)$ and initial condition is $y(-1) = 1$

Determine : (i) Zero input response
(ii) Zero State response
(iii) Total response.

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(c) A cumulative average of a signal x(n) in the interval $0 \le k \le n$ is defined as 5

$$y(n) = \frac{1}{n+1} \sum_{k=0}^{n} x(k) \qquad n = 0, 1, 2, \ldots\ldots$$

Redize this system in a recursive manner.

4. (a) Using 4 point DFT and IDFT, determine the circular convolution of 6
$x(n) = \{ 1, 2, 3, 1 \}$ and $y(n) = \{ 4, 3, 2, 2 \}$

(b) Draw FFT flow graph for N = 4 and hence find DFT for $x(n) = \{ 1, 2, 3, 4 \}$. 5

(c) $x(n) = x_1(n) + x_2(n)$. If $x(k) = 2\,Re[x_1(k)]$, find the relation between $x_1(n)$ and $x_2(n)$, when the sequences involved are real sequences. 5

(d) Let $y(n) = \begin{cases} x\left(\frac{n}{2}\right) & , \text{ n even} \\ 0 & , \text{ n odd} \end{cases}$ 4

Find 8 point y(k) in terms of x(k) without performing DFT/IDFT.

5. (a) Derive two relations to find DFT of two real N point sequences using only a single N point DFT. 4

(b) Using the relations derived in ⓐ, find DFT of the sequence : 5
$x_1(n) = \{1, 1, 1, 1\}$
$x_2(n) = \{2, 1, 2, 1\}$

(c) Find the number of real additions and real multiplications required to find DFT of a 64 point signal if – 5
(i) Direct Computation of DFT is used
(ii) DITFFT is used.

(d) Derive the relation between analog frequency variable and digital frequency variable under Bilinear transformation. Hence explain frequency warping. 6

6. (a) A digital LPF is required to meet the following specifications : 10
Passband ripple $\le$ 1 dB
Passband edge frequency z= 4 KHz
Stopband attenuation $\ge$ 40 dB
Stopband edge frequency = 6 KHz
Sampling rate = 24 KHz
The filter is to be designed using Bilinear Transformation with Butterworth approximation. Determine two filter order.

(b) Derive the expression for frequency sampling realization of FIR filter and sketch the same. 10

7. (a) Some of the zeros of a 5th order linear phase filter are located at

$$Z_1 = -1, \quad Z_2 = 0.5\, e^{+\frac{\pi}{3}}$$

(i) Complete two pole – zero diagram 2
(ii) Is the filter symmetric ? Draw its phase response to prove that it is a linear phase filter. 4
(iii) Find the transfer function of the filter and realize it using minimum number of components. 4

(b) State and prove convolution property of z-transform. 5

(c) Show pole-zero diagram for a system having transfer function $H(z) = 1 + z^{-4}$. 5