S.E.(Com) Sem III (R) 08/06/09

Con. 3335–09. (REVISED COURSE) VR–3318

Lib

(3 Hours) [Total Marks : 100

Applied Mathematics III 3 p.m. to 6 p.m.

N.B. : (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions out of remaining **six**.
(3) Non-programmable **calculator** is **allowed**.
(4) Write the **sub questions** of the main question collectively.

1. (a) Find Z-transform of $\cos\left(\frac{\pi k}{3}+\alpha\right)$, $k \geq 0$ **5**

(b) State and prove change of scale property. **5**

(c) Find A^{-1} using adjoint method where **5**

$$A=\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & 3\\ 1 & 4 & 9\end{bmatrix}$$

(d) Find the complex form of the Fourier series for $f(x)=e^{-x}$ in $(-1, 1)$. **5**

2. (a) Evaluate $\int_0^{\infty} e^{-t}\,\frac{\sin^2 t}{t}\,dt$. **6**

(b) Obtain the fourier series for $f(x) = x\cos x$ in $(-\pi, \pi)$. **6**

(c) If A, B, C are non-singular matrices of order n x n then prove that **8**

$$\begin{bmatrix}A & O\\ B & C\end{bmatrix}^{-1}=\begin{bmatrix}A^{-1} & O\\ -C^{-1}BA^{-1} & C^{-1}\end{bmatrix}$$

Hence find the inverse of $\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 2 & 0 & 0\\ 3 & 0 & 4 & 0\\ 0 & 4 & 0 & 2\end{bmatrix}$

3. (a) Using convolution Th^m find the Laplace inverse transform of **6**

$$\frac{5}{\left(s^2+a^2\right)\left(s^2+b^2\right)}$$

(b) Show that every square matrix can be uniquely expressed as the sum of symmetric and skew-symmetric matrices. **6**

(c) Find Fourier cosine integral for **8**

$$f(x) = 1-x^2, \quad 0 \leq x \leq 1$$
$$= 0, \quad x > 1$$

Hence evaluate $\int_0^{\infty}\left(\frac{x\cos x-\sin x}{x^3}\right)\cos\frac{x}{2}\,dx$.

4. (a) Using L.T. solve, $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 8y = 1$, $y(0) = 0$, $y'(0) = 1$. 6

(b) Find the non-singular matrices P and Q such that PAQ is in normal form. Also find its rank. Where 6

$$A = \begin{bmatrix} 2 & 1 & 4 \\ 3 & 2 & 2 \\ 7 & 4 & 10 \\ 1 & 0 & 6 \end{bmatrix}$$

(c) Express as a Fourier series, 8

$$\begin{aligned} f(x) &= \pi x \quad , \ 0 \le x \le 1 \\ &= \pi(2 - x) \quad , \ 1 \le x \le 2. \end{aligned}$$

5. (a) Find $L\{f(t)\}$ where $f(t) = \sin 2t$, $0 \le t \le \frac{\pi}{2}$, $f(t) = 0$, $\frac{\pi}{2} \le t \le \pi$ and $f(t) = f(t + \pi)$. 6

(b) Obtain half-range cosine series for 6

$f(x) = (x - 1)^2$ in $0 < x < 1$.

(c) Find the inverse Z–transform of $f(z) = \frac{1}{(z-3)(z-2)}$. 8

If region of convergence is (i) $|z| < 2$ (ii) $2 < |z| < 3$ (iii) $|z| > 3$.

6. (a) Show that the set of functions 6

$$\sin\left(\frac{\pi x}{2l}\right),\ \sin\left(\frac{3\pi x}{2l}\right),\ \sin\left(\frac{5\pi x}{2l}\right), \ldots\ldots \text{ is}$$

orthogonal over $(0, l)$.

(b) Find $Z\{2^k \sin(3k + 2)\}$, $k > 0$. 6

(c) Find the inverse L.T. of – 8

(i) $e^{-s}\left\{\frac{1-\sqrt{s}}{s^2}\right\}^2$ (ii) $\frac{s + 29}{(s + 4)\ (s^2 + 9)}$.

7. (a) Test the consistency of the following equations and solve them if they are consistent. 6

$x + y + z = 6$, $x - y + 2z = 5$,

$3x + y + z = 8$, $2x - 2y + 3z = 7$.

(b) Find the Fourier series for 6

$$\begin{aligned} f(x) &= 0, \quad -\pi < x < 0 \\ &= \sin x, \quad 0 < x < \pi \end{aligned}$$

Hence deduce that,

$$\frac{1}{2} = \frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \ldots\ldots$$

(c) Find the L. T. of 8

(i) $\frac{\sin t \sinh t}{t}$

(ii) $$\begin{aligned} f(t) &= \cos t, \quad 0 < t < \frac{\pi}{2} \\ &= \sin t, \quad t > \frac{\pi}{2}. \end{aligned}$$