S.E. (ETRX) Sem III (R)

Con. 3289–09. (REVISED COURSE) VR–3303

(3 Hours) [Total Marks : 100

Engineering Mathematics - III

08/06/09
3 p.m. to 6 p.m.

N.B. : (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions out the remaining **six** questions.
(3) **Figures** to the **right** indicate marks assignment.
(4) Assume **suitable** data if **required**.

1. (a) If $L\left[\text{erf}\sqrt{t}\right] = \frac{1}{s\sqrt{s+1}}$, Find $L\left[t \cdot \text{erf}\, 2\sqrt{t}\right]$. 5

(b) Obtain the expansion of $f(x) = x(\pi - x)$, $0 < x < \pi$ as a half range cosine series. 5

Hence show that $\sum_{1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.

(c) Find the bilinear transformation which maps the points $z = \infty$, i, 0 onto the pts $w = 0$, i, ∞. 5

(d) Evaluate $\int_c \bar{F} \times d\bar{r}$ where. 5

$\bar{F} = (2xy + z^2)i + x^2j + 3xz^2k$ along the curve $x = t$, $y = t^2$, $z = t^3$ from (0, 0, 0) to $(1, 11)^5$.

2. (a) Prove that following Matrices is orthogonal – 6

$$A = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$$ hence find A^{-1}

(b) Find inverse Laplace transform of $\frac{(s-1)^2}{(s^2 - 2s + 5)^2}$ using convolution theorem. 6

(c) Show that $u = e^x(x \cos y - y \sin y)$ is harmonic. Find the harmonic conjugate v and analytic function f(z). 8

3. (a) Find the Fourier series for the following function – 6

$$f(x) = \frac{3x^2 - 6x\pi + 2\pi^2}{12} \text{ in } (0, 2\pi).$$

(b) Find z-transform of $\{ 2^x \sin(3k + 2) \}$, $k \geq 0$. 6

(c) Verify Green's Theorem in the plane for $\int_c (xy + y^2)\, dx + x^2\, dy$ where c is closed curve of the region bounded by $y = x$ and $y = x^2$. 8

4. (a) Obtain the complex form of Fourier series for $f(x) = e^{ax}$ in $(0, a)$. 6

(b) Reduce the following matrix to normal form and hence find it's rank 6

$$\begin{bmatrix} 1 & 2 & -2 & 3 & 1 \\ 1 & 3 & -2 & 3 & 0 \\ 2 & 4 & -3 & 6 & 4 \\ 1 & 1 & -1 & 4 & 6 \end{bmatrix}$$

(c) Find Laplace transform of the following –

(i) $\dfrac{e^{-2t} \sin 2t \cosh t}{t}$ 4

(ii) $\int_0^t u \cdot e^{-3u} \sin^2 u \, du$. 4

5. (a) Find Laplace transform of 6

$f(t) = \text{Sin } 2t \quad 0 < t < \pi/2$

$= 0 \quad \pi/2 < t < \pi$

and $f(t) = f(t + \pi)$.

(b) Verify Stokes theorem for $\overline{F} = x^2 i + xy j$ where c is the boundary of the rectangle $x = 0, y = 0, x = a, y = b$. 6

(c) Find non-singular matrices P and Q. Such that 8

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$$ is reduced to normal form .

Find it's rank and A–1.

6. (a) Find the inverse z-transform of 8

$$\frac{1}{(z-2)(-z-3)} \quad |z| < 2, \quad 2 < |z| < 3, \quad |z| > 3.$$

(b) Find the Fourier cosine transform of $f(x) = e^x + e^{-2x}$ $(x > 0)$. 6

(c) Find the image of imaginary axis in z-plane on w-plane under the transformation 6

$$w = \frac{1}{z+i}.$$

7. (a) For what value of λ the equation 6

$$x + y + z = 1$$
$$x + 2y + 4z = \lambda$$
$$x + 4y + 10z = \lambda^2$$

have a solution and solve then completely in each case.

(b) Solve by using Laplace transform 6

$(D^2 + 2D + 5)y = e^{-t} \sin t$ when $y(0) = 0 \quad y'(0) = 1$.

(c) Express the function $f(x) = \begin{cases} \sin x & 0 < x \leq \pi \\ 0 & x < 0, x > \pi \end{cases}$ 8

as Fourier integral and prove that

$$f(x) = \frac{1}{\pi} \int_0^\infty \frac{\cos w x + \cos [w (\pi - x)]}{1 - w^2} \, dw$$